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Newton'un Soğuma Yasası Denkleminin Kashuri Fundo Dönüşümü ile Analitik Çözümü

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ÖZET

Geçmişte olduğu gibi günümüzde de fiziksel olayların anlaşılması, doğru bir şekilde yorumlanabilmesi ve modellenmesi gelişmiş matematiksel yöntemlerin kullanılmasını gerektirir. Bu bağlamda, Newton'un soğuma yasası gibi ısı transferi problemlerinin çözümü, integral dönüşümü gibi güçlü matematiksel araçlarla karmaşık hesaplamalara gerek kalmadan, doğru, güvenilir ve kolaylıkla elde edilir. Newton'un soğuma yasası bir cismin sıcaklığının çevresel sıcaklıkla nasıl etkileşime girdiğini ve zaman içinde nasıl değiştiğini diferensiyel denklem modelleriyle ifade eder. Değişkenler ve değişim hızları arasındaki karmaşık ilişkileri ifade eden bu denklemler, fizikçilerin kesin matematiksel modeller formüle etmelerine olanak tanıyarak, fiziksel sistemlerin davranışlarına ilişkin doğru yorumlar yapılmasını sağlarlar. Diferensiyel denklemlerin çözümlerini elde etmeye yönelik hesaplamalar, cebirsel denklemlere ilişkin hesaplamalardan daha karmaşık olabilir. Bundan dolayı, bu denklemlerin çözümlerini elde etmek için farklı yöntemler kullanılmıştır. Bu makalede, Newton'un soğuma yasasının integral dönüşümlerinin bir çeşidi olan Kashuri Fundo dönüşümü ile çözümünü ve bu yaklaşımın fizik, biyokimya, ekonomi, finans, mühendislik vb. alanlarda yer alan farklı matematiksel modellerin çözümlerine ulaşmada kullanılabilecek etkili ve güvenilir bir yöntem olduğunu ortaya koyuyoruz.

Analytical Solution of Newton's Law of Cooling Equation via Kashuri Fundo Transform

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ABSTRACT

As in the past, understanding, correctly interpreting and modeling physical phenomena requires the use of advanced mathematical methods. In this context, the solution of heat transfer problems such as Newton's cooling law is obtained accurately, reliably and easily without the need for complex calculations with powerful mathematical tools such as integral transform. Newton's law of cooling expresses how the temperature of a body interacts with the environmental temperature and changes over time by differential equation models. These equations, expressing the complex relationships between variables and rates of change, provides accurate interpretations of the behavior of physical systems by allowing physicist formulating precise mathematical models. Calculations to obtain solutions of differential equations can be more complex than calculations for algebraic equations. Therefore, different methods have been used to get the solutions of these equations. In this article, we present the solution of Newton's cooling law with Kashuri Fundo transformation, which is a type of integral transformations, and that this approach is an effective and reliable method that can be used to reach solutions of different mathematical models in the fields of physics, biochemistry, economics, finance, engineering, etc.

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INTRODUCTION

Every event that takes place in the universe has a mathematics. Scientists reveal the mathematics of the rules behind the events using certain models [1]. Thanks to these models, the events taking place in the universe can be understood and interpreted correctly.

Since most of the events in the universe involve change, differential equations are often used to model these events. Because mathematically, rates of change are defined by derivatives, and since differential equations are equations containing derivatives of one variable with respect to another variable, they provide us with great convenience in understanding and interpreting change.

Differential equations are used to model important events in many different fields such as population growth of species, change in return on our investments, radioactive decay, interest problems, cancer growth and spread of diseases in medicine, finding optimum investment strategies in economics, cooling and heating problems in physics, and many more [2]. Differential equations are important for accurately expressing the foundation of modern physics because they play an important role in understanding and interpreting the complex rules of the natural world. They provide mathematical language for expressing and interpreting the fundamental principles underlying physical phenomena that extend from classical mechanics to beyond quantum mechanics. Differential equations enable us to model and interpret complex real-life problems and reveal the fundamental relationships that underlie them.

Newton's law of cooling, which has a very important position in physics, is also modeled by differential equations. This law explains how the temperature change in an object occurs depending on the difference between the temperature of the object and the temperature of the environment in which the object is located. Newton's law of cooling states that a hot body releases heat energy into its environment [3-5]. The amount of this energy emitted depends on the temperature difference between the object placed in the environment and the environment. While this energy transfer is taking place, as the temperature difference decreases, the energy transfer decreases and eventually the hot body equalizes with the ambient temperature [3-5]. Newton's law of cooling is generally modeled by the linear ordinary differential equation [3]

$$\frac{dT(t)}{dt} = -C(T(t) - T_e) \tag{1}$$

with the initial condition

$$T(t_0) = T_0$$

where T is the temperature of the substance, T_e is the constant temperature of the environment, T_0 is the initial temperature of the substance at time t_0 and C is the proportionality constant, known as convective heat transfer coefficient. This law is used in many different application areas. For example, in engineering, it is the basis for temperature control and heat transfer issues. In addition to this, we encounter this law in our daily lives, for example, in matters such as thermal insulation, air conditioning systems and energy efficiency in our homes and vehicles. It also helps us understand how the temperatures of stars and planets change in astronomy and meteorology. As a result, Newton's law of cooling is a valuable tool for scientists in the development of modern technology, as it forms the basis of many applications related to energy transfer and temperature control.

The variety of uses of differential equations and the modeling of important phenomena in physics make it important to reach the solutions of such equations. In the literature, there are many methods that can be used to reach solutions of differential equations [6-9]. In this study, we are looking for a solution to Newton's law of cooling, which has a very important place in physics, by using Kashuri Fundo transform [10].

The basis of the Kashuri Fundo transform is the logic of expressing some properties of this function in a more easily analyzable form by taking a certain transformation of a function, as in other integral transformations. They often transform complex differential equations or functions into simpler mathematical operations, making the solution of these equations or the analysis of functions easier [10]. Since the equations obtained as a result of the transformation usually have a more standardized and easier to understand form, the analytical or numerical solutions of the equations become easier and more general. There are many different studies in the literature that reveal the accuracy of these statements [11-21]. There are many studies that show that it gives effective results when used by blending with different methods to reach solutions of nonlinear and fractional differential equations [22-32]. In this study, we demonstrate that the Kashuri Fundo transform, based on Newton's cooling law, which is modeled with first-order differential equations, is an effective, reliable and time-saving method for reaching solutions of first-order differential equations.

MATERIALS AND METHODS

Kashuri Fundo Transform

Definition 1. [10] We consider functions in the set F defined as

$$F = \left\{ f(x) \mid \exists M, m_1, m_2 > 0 \quad \text{such that} \quad |f(x)| \leq M e^{\frac{|x|}{m_i^2}}, \quad \text{if} \quad x \in (-1)^i \times [0,\infty) \right\}$$

For a function in the set defined above, M must be finite. m_1, m_2 may be finite or infinite.

Definition 2. [10] Kashuri Fundo transform defined on the set F and denoted by $K(.)$ is defined as

$$K[f(x)](v) = A(v) = \frac{1}{v}\int_0^\infty e^{\frac{-x}{v^2}} f(x)dx, \quad x \geq 0, \quad -m_1 < v < m_2$$

Inverse Kashuri Fundo transform is expressed as: $K^{-1}\left[A(v)\right] = f(x), \quad x \geq 0$.

Theorem 1 (Sufficient Conditions for Existence). [10] If $f(x)$ has exponential order $\frac{1}{k^2}$ and is piecewise continuous on $[0,\infty)$, then $K[f(x)](v)$ exists for $|v| < k$.

Theorem 2 (Linearity Property). [10] Let the Kashuri Fundo transforms of $f(x)$ and $g(x)$ exist and c_1, c_2 are scalar. Then

$$K[(c_1 f \pm c_2 g)(x)] = c_1 K[f(x)] \pm c_2 K[g(x)]$$

Theorem 3 (Transformation of the Derivatives). [10] Let's assume that the Kashuri Fundo transform of $f(x)$, denoted by $A(v)$, exists. Then

$$K[f^{(n)}(x)] = \frac{A(v)}{v^{2n}} - \sum_{k=0}^{n-1} \frac{f^{(k)}(0)}{v^{2(n-k)-1}} \tag{2}$$

Table 1. *Kashuri Fundo Transform of Some Special Functions [10,27]*

f(x)	**A(v)**
1	v
x	v^3
x^n	$n!v^{2n+1}$
e^{cx}	$\frac{v}{1-cv^2}$
$\sin(cx)$	$\frac{cv^3}{1+c^2v^4}$
$\cos(cx)$	$\frac{v}{1+c^2v^4}$
$\sinh(cx)$	$\frac{cv^3}{1-c^2v^4}$
$\cosh(cx)$	$\frac{v}{1-c^2v^4}$
x^α	$\Gamma(1+\alpha)v^{2\alpha+1}$

APPLICATION

Kashuri Fundo Transform of Newton's Law of Cooling

In this section, we will apply Kashuri Fundo transform to Newton's law of cooling equation and examine the effectiveness of this method through some numerical applications.

First, taking the Kashuri Fundo transform of the equation (1), we obtain

$$K\left[\frac{dT(t)}{dt}\right]=K\left[-C(T(t)-T_e)\right] \tag{3}$$

Now, rearranging the equation (3) according to the equation (2) and substituting the value in the initial condition into this equation, we get

$$\frac{A(v)}{v^2}-\frac{T_0}{v}=-C\left[A(v)-T_e v\right] \tag{4}$$

where $A(v)=K\left[T(t)\right]$.

If we make the equation (4) suitable for applying the inverse Kashuri Fundo transform, we get

$$A(v)=T_e v-T_e\frac{v}{1+cv^2}+T_0\frac{1}{1+cv^2} \tag{5}$$

Finally, applying the inverse Kashuri Fundo transform to the equation (5) using table 1, we will find the solution of Newton's law of cooling equation as

$$T(t)=T_e+(T_0-T_e)e^{-ct}.$$

Application 1. [33] A hot milk with initial temperature $115°\ C$ is kept in an environment with temperature $35°\,C$. Since the rate of temperature change is $20°\ C$ per/min, how long will it take for this milk to cool down to temperature $40°\ C$?

Assuming that milk obeys Newton's law of cooling, we arrange the equation (1) according to the values given in the question as

$$\frac{dT(t)}{dt} = -C(T-35) \tag{6}$$

with the initial conditions

$$T(0) = 115, \quad T'(0) = -20 \tag{7}$$

First, using the initial conditions expressed in (7) in the equation (6), we find the value of C as

$$\begin{aligned} -20 &= -C(115-35) \\ C &= 0.25 \end{aligned} \tag{8}$$

Substituting this C value we found in the equation (6), we get

$$\frac{dT(t)}{dt} = -0.25(T-35) \tag{9}$$

Now, applying the transform to both sides of the equation (9), we obtain

$$K\left[\frac{dT(t)}{dt}\right] = -0.25K\left[T-35\right] \tag{10}$$

Rearranging the equation (10) according to the equation (2) and table 1, we get

$$\frac{A(v)}{v^2} - \frac{115}{v} = -0.25\left[A(v) - 35v\right] \tag{11}$$

Rearranging this equation, we get

$$A(v) = \frac{115v}{1+0.25v^2} + 35v - \frac{35v}{1+0.25v^2} \tag{12}$$

Finally, applying the inverse Kashuri Fundo transform to the equation (12), we find the solution of equation (6) as

$$T(t) = 80e^{-0.25t} - 35 \tag{13}$$

Now we can find out how long it will take for the milk to cool down to $40°\ C$ based on our solution. For this, we organize the solution as

$$40 = 35 + 80e^{-0.25t} \tag{14}$$

If we solve this equation, we find the time we are looking for as

$$\begin{aligned} 80e^{-0.25t} &= 5 \\ e^{0.25t} &= 16 \\ 0.25t &= \ln 16 \\ t &= 11.090354889 \end{aligned}$$

This result is in good agreement with the results obtained by other methods [33-35].

Application 2. [33] The heated iron with an initial temperature of $50°\ C$ is kept in an environment with temperature of $27°\ C$. Since the rate of temperature change is $3°\ C$ per/min, how long will it take for this iron to cool down to temperature $36°\ C$?

Assuming that iron obeys Newton's law of cooling, we arrange the equation (1) according to the values given in the question as

$$\frac{dT(t)}{dt} = -C(T-27) \tag{15}$$

with the initial conditions

$$T(0) = 50, \quad T'(0) = -3 \tag{16}$$

First, using the initial conditions expressed in (16) in the equation (15), we find the value of C as

$$\begin{aligned} -3 &= -C(50-27) \\ C &= 0.13 \end{aligned} \tag{17}$$

Substituting this C value we found in the equation (15), we get

$$\frac{dT(t)}{dt} = -0.13(T-27) \tag{18}$$

Now, applying the Kashuri Fundo transform to both sides of the equation (18), we obtain

$$K\left[\frac{dT(t)}{dt}\right] = -0.13K\left[T-27\right] \tag{19}$$

Rearranging the equation (19) according to the equation (2) and table 1, we get

$$\frac{A(v)}{v^2} - \frac{50}{v} = -0.13\left[A(v) - 27v\right] \tag{20}$$

Rearranging this equation, we get

$$A(v) = \frac{50v}{1+0.13v^2} + 27v - \frac{27v}{1+0.13v^2} \tag{21}$$

Finally, applying the inverse Kashuri Fundo transform to the equation (21), we find the solution of equation (15) as

$$T(t) = 23e^{-0.13t} + 27 \tag{22}$$

Now we can find out how long it will take for the milk to cool down to $36°\ C$ based on our solution. For this, we organize the solution as

$$36 = 27 + 23e^{-0.13t} \tag{23}$$

If we solve this equation, we find the time we are looking for as

$$\begin{aligned} 23e^{-0.13t} &= 9 \\ e^{0.13t} &= \frac{23}{9} \\ 0.13t &= \ln\frac{23}{9} \\ t &= 7.2175 \end{aligned}$$

This result coincides with the ones found by other methods [33-35].

Application 3. [36] While the ambient temperature is $20°C$ the temperature of the water drops from $100°C$ to $80°C$ in 20 minutes. What will be the temperature after 30 minutes and how long will it take for this water to cool to $45°C$?

Assuming that water obeys Newton's law of cooling, we arrange the equation (1) according to the values given in the question as

$$\frac{dT(t)}{dt} = -C(T-20) \tag{24}$$

with the initial conditions

$$T(0)=100, \quad T(20)=80 \tag{25}$$

First, applying the Kashuri Fundo transform to both sides of the equation (24), we obtain

$$K\left[\frac{dT(t)}{dt}\right] = -CK\left[T-20\right] \tag{26}$$

Rearranging the equation (26) according to the equation (2) and table 1, we get

$$\frac{A(v)}{v^2} - \frac{100}{v} = -C\left[A(v)-20v\right] \tag{27}$$

Rearranging this equation, we get

$$A(v) = \frac{80v}{1+Cv^2} + 20v \tag{28}$$

Finally, applying the inverse Kashuri Fundo transform to the equation (28), we find the solution of equation (24) as

$$T(t) = 80e^{-Ct} + 20 \tag{29}$$

Now let's find the value of C . If we use $T(20)=80°$ in the equation (29), we get

$$\begin{gathered} 80 = 20 + 80e^{-20C} \\ e^{-c} = \left(\frac{3}{4}\right)^{\frac{1}{20}} \end{gathered} \tag{30}$$

Using the equation (30) to find $T(30)$, we obtain

$$\begin{gathered} T(30) = 20 + 80e^{-30c} \\ T(30) = 20 + 80\left(\frac{3}{4}\right)^{\frac{3}{2}} = 71.96°\ C \end{gathered} \tag{31}$$

Now we can find out how long it will take for the water to cool down to $45°\ C$ based on our solution. For this, we organize the solution as

$$45 = 20 + 80(e^{-C})^{t}$$

$$\left(\frac{3}{4}\right)^{\frac{t}{20}} = \frac{25}{80}$$

$$t = 80.8636283723$$

This result is in good agreement with the result obtained by other method [36].

DISCUSSION AND CONCLUSIONS

Our aim in carrying out this study is to show that Kashuri Fundo integral transform is an effective, reliable, time-saving method that does not require complex calculations in reaching solutions of first-order ordinary differential equations. In order to show, Newton's law of cooling equation, which is modeled by first-order ordinary differential equation, was used in the application of the transform. After applying Kashuri Fundo transform to the differential equation model expressing Newton's law of cooling, reinforcement was made on three different numerical examples to reveal the usefulness of the method. As a result of these applications, it has been concluded that Kashuri Fundo method is an effective, reliable and simple calculation method that can be used in the solutions of differential equations. Based on this result, it can be said that this method can provide convenience to scientists in reaching the solutions of most real-life problems modeled by existing or being developed differential equations.

Conflict of Interest

The authors have no conflicts of interest to disclose for this study.

Authorship Contribution Statement

B.P.: Methodology, Writing - Review & Editing, **F.A.Ç.:** Formal Analysis, Investigation, Methodology, Writing - Original Draft, **H.A.P.:** Supervision, Conceptualization, Investigation, Writing - Review & Editing

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