TECHNOLOGY SUPPORT FOR LEARNING EXPONENTIAL AND LOGARITHMIC FUNCTIONS

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Abstract

This study aims to examine the extent to which the mobile application of Desmos graphing calculator supports undergraduate students' learning of exponential and logarithmic functions at the Middle School Mathematics Education Program in Faculty of Education. More specifically, the study investigates the undergraduate students' views about and actions in utilizing Desmos while learning exponential and logarithmic functions. Convenience and purposive sampling methods were used to conduct this study. Seventeen freshmen were participated to the study within the context of Fundamentals of Mathematics course where the exponential and logarithmic functions were introduced to the undergraduates and in which one of the researchers was the instructor. Following the qualitative research principles, case study design was conducted to collect data from observation and documental sources. The data come from the observations of students' in-class activities, classroom discussions, researchers' field notes, and reflection papers over a 3-week period that was scheduled to teach the concept of exponential and logarithmic functions. The content analyses of the data reveal that undergraduate students find Desmos graphing calculator beneficial by highlighting its affordances such as i) compensating the lack of procedural knowledge, ii) providing opportunities for exploration, and iii) enhancing engagement with the tasks. Thus, the study shows that Desmos is a multipurpose learning source for learning exponential and logarithmic functions. Finally, the study discusses the role of Desmos on learning functions and provides implications for its use in undergraduate mathematics courses.

Keywords: Educational technology; Desmos graphing calculator; exponential function; logarithmic function; undergraduate students

INTRODUCTION

The affordances of the educational technologies are multi-faceted. They allow students to engage in mathematics through exploration of the content, reasoning for intentional actions on the tools, questioning the processes of the tool and students' own reasoning, interacting with the technology and deciding on what to do (Karadeniz & Thompson, 2017). Moreover, conceptual understanding of mathematical topics can be supported with the use of technology (Liang, 2016), and procedural knowledge can be interpreted within a technological context. In fact, the use of educational technologies in undergraduate mathematics courses has been given great importance in learning the fundamental mathematical topics (Oates, Sheryn, & Thomas, 2014) and in helping students find quick and easy ways to discover the mathematical topics under the guidance of teachers (Hoang & Caverly, 2013).

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Teachers are aware of the fact that graphing calculators provide students with the opportunity to investigate mathematical facts through visualization and dynamicity by giving time for exploration in their lessons (Karadeniz & Thompson; 2017). Desmos program is a graphing calculator application with a free and online/offline mode for using the application which requires only basic technological skills (Thomas, 2015). Additionally, the program can be used on tablets or smartphones not only as a graphing calculator but also as an application that enables users to download previously designed activities (Desmos, 2015; Edwards, 2015). Learners in the related studies find it user friendly (Oates, Sheryn, & Thomas, 2014). Besides, one can access the screen size of the device while using the application and it is fast in graphing (King, 2017).

Functions, which constitutes one of the fundamental subjects of mathematics, is a unifying concept that makes the interconnections of mathematical concepts visible (Argün, Arıkan, Bulut, & Halıcıoğlu, 2014). Function concept and function types are considered to have a central role in mathematics, especially in algebra and calculus (Carlson, Oehrtman, & Engelke, 2010). It is important for students to comprehend the topic because it is taught in mathematics courses in each grade level (Biehler, Scholz, & Winkelman, 1993).

Exponential and logarithmic functions in school mathematics include the relations between exponential functions and logarithm, and the meaning of variable and constant as mathematical concepts taught within real-life situations (Çetin, 2004). The meaning of the symbols or notations denotes mathematics as a special language (Weber, 2002a). Specifically, for logarithmic functions, students have difficulty in interpreting the graphs (e.g., Ural, 2017), understanding the meaning of the notation (e.g., Zazkis, 2006), and interpreting those functions regarding operations (e.g., Weber, 2002a; Weber, 2002b). According to Hurwitz (1999), "students often have difficulty in thinking of a logarithm as the output of a function because the notation used for logarithms does not look like the familiar $f(x)$ notation" (p. 334) indicating that the relationship between those functions is not so easy to be comprehended by students (Gramble, 2005). They need to deal with different mathematical concepts to capture the meaning of logarithm. The areas where students have difficulties can be categorized as 'logarithms as inverse functions', 'logarithmic notation', and 'unique properties of logarithms' (Kenney & Kastberg, 2013; p. 17). These difficulties in each category show that learners need to understand which notations for logarithmic functions are used, how those notations can be considered together with prior experiences about variable in an expression, and why students' reasoning of those functions cannot be done in the way they think.

Inverse functions, which explain the relationship between the exponential and the logarithmic functions, cannot be easily comprehended. For instance, students tend to use a calculation process of reversing x and y, and the function is solved for y as they were once taught in high school. In addition to this, even if the tabular representation of the expressions is shown correctly, they may be misrepresented as a linear graph (Ural, 2006). According to Weber (2002a), students have difficulty in understanding exponents and logarithms as a process, and it is necessary that students perceive those functions as process-object duality in order to understand calculus and advanced mathematics. Hence, teachers intend to improve students' understanding by reasoning and forming accurate images of logarithmic functions rather than memorizing (Kenney & Kastberg, 2013). Kenney and Kastberg (2013) state that students' understanding of logarithmic functions is based on understanding of two descriptions of the logarithm; 'logarithm as an exponent' and 'logarithm as a function' (p. 13). Therefore, the exploration that these two functions are the inverse of each other is necessary for solving problems about these functions (Kenney & Kastberg, 2013).

It is equally important for teachers to design instructions by considering these needs of the students considering their possible misunderstandings or lack of understandings of the functions. To overcome these challenges, there is a number of mathematical software (i.e., GeoGebra, Desmos, and Cabri) that enable students to make mathematical explorations of functions (Hoang & Caverly, 2003; Venturini, 2015). One of the most effective software programs supporting students to learn functions is Desmos graphing calculator, which is a widely used technological tool (Liang, 2016). Smith and Shotsberger

(1997) indicate that graphing calculators that help students in different mathematical topics have been beneficial for students mostly in terms of exponential and logarithmic functions. In an experimental study, for instance, Hollar and Norwood (1999) found that undergraduate students' understanding of functions were positively affected by their use of graphing calculators for 'modeling a real-world situation, interpreting a function in terms of a realistic situation, translating among different representations of functions, or reifying functions' (p. 22). An extended examination of the relevant literature shows that Desmos graphing calculator is a useful and easy-to-use program that college-level students find enjoyable while graphing (Oates, Sheryn, & Thomas, 2014).

For teaching secondary school mathematics, it is suggested that students' cognitive conflicts can be used as learning opportunities for meaningful understanding of some concepts (e.g., limit concept) while interpreting related graphs via Desmos (Liang, 2016). To illustrate, a recent study conducted by King (2017) showed that Desmos enhanced high school students' learning of linear, quadratic, and absolute value functions. In other words, it enabled the students to describe the distinction among these functions, interpret the restrictions of domain and range of those functions, and identify the movements of functions on axes, the directions of the graphs, and the changing domains of those functions. Similarly, Bourassa (2014) suggested the use of the activities that are available on Desmos website to learn mathematics with multiple solutions so that technology can be integrated into the solutions of linear, quadratic and exponential functions, and solutions can be checked via Desmos. In this regard, Desmos graphing calculator is a facilitator for meaningful learning of functions (Karadeniz & Thompson, 2017).

The context of the study

In the Middle School Mathematics Education Program, as a graduation requirement to be a middle school mathematics teacher, the undergraduate students take pedagogical and mathematics field courses to be awarded a bachelor's degree in Middle School Mathematics Education. They broaden and build up their mathematical knowledge each academic term and acquire a higher level of mathematics knowledge as determined by the Council of Higher Education and the universities. In this study, Desmos has been incorporated to the undergraduate mathematics course in order to reveal its impacts on teaching and learning of the exponential and logarithmic functions in a technology integrated learning environment. Desmos was chosen as an educational technology tool because it has a user-friendly interface, related menus for functions, and a free downloadable feature with an offline use option. Furthermore, it can be downloaded and used on mobile phones as well. These features are vital for a technological tool to be selected as an educational tool for in-class teaching and exercises by the educators. One of its advantages is that it does not require any preliminary preparation. Besides, among other similar programs, these features all together are indigenous to Desmos.

This study emerged from the two major problems reported in previous studies about the undergraduate students' conceptual deficiencies in the functions subject. First of all, they are unable to construct graphs of exponential functions and logarithmic functions in Calculus courses (Hollar & Norwood, 1999). Second, they are having difficulties in stating algebraic and graphical expressions of the inverse relationship between these functions (Williams, 2011). Building on the above mentioned difficulties in learning exponential and logarithmic functions, Desmos graphing calculator was chosen as the main instructional tool to remedy these challenges and enhance undergraduate students' conceptual understanding and relating of these two types of functions. The study focused on how and why undergraduate students integrate Desmos graphing calculator in their learning of exponential and logarithmic functions as an educational tool. Since the related literature showed that students are profoundly affected by what they learn through the use of Desmos graphing calculator, the software can have a significant potential to support the investigation of students' learning on exponential and logarithmic functions. Additionally, students' reflections on their actions are essential to ascertain affordances and constraints of the software for mathematics educators' future applications of Desmos. The aim of this study was to reveal how undergraduate students in Middle School Mathematics Education Program use Desmos graphing calculator in their learning of exponential and logarithmic

functions. To this end, the following research question was formulated. What are the views of undergraduate students about the use of Desmos graphing calculator in learning exponential and logarithmic functions?

METHOD

In this section, research design, participants, data collection processes, and data analyses are reported in detail.

Research Design

The study was conducted by using qualitative research design. Among the qualitative research strategies, the case study design allows researchers to explore phenomena such as cases, individuals, and implementation processes within its original context through the triangulation of multiple data sources (Creswell, 2013; Yin, 2013). In this respect, the case study research was designed in order to carry out a detailed and reliable investigation of undergraduate students' practices in and views about learning exponential and logarithmic functions with the support of Desmos graphing calculator.

Design principles for teaching exponential and logarithmic functions.

The research was conducted within the Fundamentals of Mathematics course in the three-week period where exponential and logarithmic functions are scheduled to be taught. The course covers a content specific for Middle School Mathematics Education Program. One general goal of this course is to educate the students as prospective mathematics teachers who know the meaning of the function types and their components in order to draw the graphs of the functions as well as solve problems about exponential and logarithmic functions. Additionally, it is essential for undergraduate students to understand and interpret those graphs of functions and relate them to each other to make sense of fundamentals of mathematics. Based on the observations of the first researcher of this study as the instructor of the course, the undergraduate mathematics students schematize the functions as different concepts or express the connection among the related concepts about functions inadequately. Therefore, the Fundamentals of Mathematics course was designed in such a way that the undergraduate students can have the opportunity to review and build on their prior knowledge and smooth away their existing problems in conceptual understanding. With these in mind, the course was designed based on the following four principles.

i. Using tasks that support conceptual and procedural understanding in the instructional package for teaching exponential and logarithmic functions.

The studies in the related literature have documented that mathematics teachers need to have a good command of the subject matter knowledge; that's, they should know more than they will tell (Shulman, 1986; Ball, 2000). Otherwise, the lack of teacher knowledge and understanding of mathematics would result in lack of student understanding and it would run the risk of causing misunderstanding. Likewise, as prospective mathematics teachers, undergraduate students' subject matter knowledge should be essentially built on solid basis in order to have constructed mathematical knowledge. The conflicts and difficulties that undergraduate students experience in the training process provide mathematics educators with the necessary information about their needs in mathematics knowledge. Rock and Brumbaugh (2013) aimed to investigate these needs of the undergraduate students and raised a question to reveal what it means to continue understanding of mathematical concepts without thinking of the meaning of a procedure. It is necessary to answer the question by indicating that there should be a balance between the conceptual understanding and the application of procedural information (Weber, 2002a; Williams, 2011) since conceptual and procedural knowledge are essential components to facilitate a meaningful learning process for the students. As it is understood from the following definition, conceptual knowledge of any concept illustrates multiple established connections among interrelated concepts:

> *'... knowledge that is rich in relationships. It can be thought of as a connected web of knowledge, a network in which the linking relationships are as prominent as the discrete pieces of information. Relationships pervade the individual facts and propositions so that all pieces of information are linked to some network' (Hiebert & Lefevre, 1986, pp. 3-4).*

The students need to be guided to explore a mathematical concept throughout the learning process through establishing connections since they can effectively learn by means of linking previous knowledge to the newly discovered. In addition to the conceptual knowledge, the students need to be procedurally competent while they are solving problems. In other words, conceptual knowledge is not sufficient to come to the conclusion that the students possess necessary amount of knowledge in the subject matter. One needs to be equipped with procedural knowledge as well, which can be defined in two parts as in the following:

> *'one part is composed of the formal language, or symbol representation system of mathematics. The other part consists of the algorithms, or rules, for completing mathematical tasks' (Hiebert & Lefevre, 1986, p. 6).*

Weber (2002a) also proposes that students need to have procedural and conceptual knowledge for exponential and logarithmic functions. However, it is also essential to determine how the students are provided with this knowledge. While exponents are taught to the students as repeated addition, they might have limited understanding of 2^{-1} or $2^{1/2}$ (Weber, 2000b). In line with this, it is suggested that it is necessary to consider exponentiation as a process, and in return, exponential and logarithmic functions need to be viewed 'as a result of applying this process' (Weber, 2000b, p. 1020). This is also supported by Williams (2011) who asserts that logarithms should be understood as objects, as processes, as functions and within contextual problems.

Using the statements above as a springboard, it can be stated that there are stages to be accomplished to develop the understanding of exponents and logarithms. In order to comprehend exponential functions, the following stages need to be completed successfully: 'exponentiation as an action', 'exponentiation as a process', 'exponential expressions as the result of a process', and 'generalization'. When students conceptualize exponents as a process, they can have an understanding of the increase of 2^x for every positive integer, and they can also comprehend that the verbal expression of the question "x is the product of how many factors of b?" is $\log_b x$ (Weber, 2002b, p.5). It is necessary for the students to understand that this generalization can be drawn for other number sets such as fractions, negative numbers, and irrational numbers as in natural numbers. What's more, students have difficulty in explaining the role of x within the function of $f(x)= a^x$, and they need to answer the questions that require understanding the reasons behind the procedural knowledge about the exponential and logarithmic functions. Moreover, Weber (2002b) summarizes that for meaningful understanding of exponential numbers, students need to comprehend multiple meanings of the number: operation, mathematical structure, and function. While operational understanding is related to repeated multiplication of an exponential number, structural understanding requires to consider it as a number that is the product of x factors of b and as the number of factors of b that are in the number m' (p.1020).

Aligned with the purpose of enhancing students' procedural and conceptual knowledge in exponential and logarithmic functions, the instructional package was in the form of a booklet prepared by and utilized in Çetin's study (2004). The booklet was basically composed of 4 parts: (1) Exponential functions, (2) Transition from exponential functions to logarithmic functions, (3) Logarithmic functions, and (4) Real life problems. Parts 1, 2 and 3 were designed with the same sequence of instructions with a linear approach, and the last part included five linearly organized tasks on exponential and logarithmic functions using real life situations and applications. For instance, paramecium production problem was used for exponential growth. The students learnt and practiced comparison of these two function types with respect to differing bases, examined the relationship between them, solved problems about the algebraic expressions of these functions, and interpreted their graphs. More

specifically, the tasks in the booklet required the students to model exponential and logarithmic functions within different real-life situations, transfer data into tables, express the data mathematically, calculate values, and draw graphics of the functions. Moreover, they consisted of questions requiring examinations of the values that exponential and logarithmic functions can have with different bases, the location of these values on the graph, and determination of the domain and range sets of the functions. The students were also expected to make sense of the concepts of increasing and decreasing functions and the symmetrical relation between them according to $y=x$, $x=0$, and $y=0$ lines. Besides, after having completed these tasks, the students could understand that the exponential function and logarithmic function are two different forms of the same function. At the end of the first part, the reverse of the exponential function was questioned in order to evoke the need for logarithmic functions.

The researchers designated goals for the three-week implementation of the booklet. These goals were driven from Higher education qualifications prepared in accordance with the learning objectives of the course and the objectives of high school level curriculum. At the end of the implementation, upon completing all of the tasks with didactic instructions in the booklet, the undergraduate students were expected to be able to:

1. Understand and interpret mathematical definition of exponential and logarithmic functions
2. Write the expression of exponential and logarithmic functions
3. Draw and interpret the graph of exponential and logarithmic functions
4. Interpret exponential functions with different positive integer bases; $f(x) = a^x$ where $a > 1$.
5. Interpret exponential functions with different rational number bases; $f(x) = a^x$ where $0<a<1$.
6. Interpret logarithmic functions with different positive integer bases; $f(x) = \log_a x$ where $a > 1$.
7. Interpret logarithmic functions with different rational number bases; $f(x) = \log_a x$ where $0<a<1$.
8. Interpret logarithmic functions with the base of e; $f(x)= \log_e x$.
9. Recognize and interpret the domain and range sets of exponential and logarithmic functions.
10. Express the relationship between exponential and logarithmic functions.

In the ninth week of the Fundamentals of Mathematics course, after functions concept was thought, the participants were asked to download the Desmos application on their mobile phones and the hard copy of the exponential and logarithmic functions instructional package was distributed as a booklet to each participant. They were asked to follow the instructions in order. The instructor guided the process by initiating and ending each task, and directed the discussions at the end of each task. Upon completing each task, the participants were directed to proceed to the next task and when everyone completed, they were asked to share their solutions within groups. Then, the solution of the task was discussed as a whole class. After deciding on the correct solution, the participants were allowed to move on to the following task. By following this cycle, the booklet was completed in three weeks, in nine course hours.

ii. Peer interaction and collaboration.

Peer interaction was considered to be a principal requirement for the implementation of the instructional package. The students were supposed to express their ideas, comment on each other's answers, help each other sort out their misunderstandings, benefit from peer corrections for effective classroom learning in technology supported classroom atmosphere (Liang, 2016). Students are likely to have some common learning experiences about the exponential and logarithmic functions, and those experiences can lead to learning opportunities through peer interaction. When functions are taken into consideration in the level of undergraduate mathematics courses, students' challenges and difficulties can be remedied through peer discussions and collaborative work on problematic issues of students (Caniglia, Borgerding & Meadows, 2017; Sfard, 1992). Considering these advantages, a collaborative classroom environment was formed in the implementation process. To achieve this, the participants were asked to work in groups of 3 or 4, engage in group discussions upon completing each task, exchange ideas about different answers within the group, decide on one single group answer, and report the group's idea in classroom discussion.

iii. The role of the instructor.

The instructor of the Fundamentals of Mathematics course - the first author of the study - collected the data through observation, classroom discussions and her field notes. Hence, being in the participant-observer role provided the researcher with the data for the study. In this role as the instructor of the course, guiding questions were asked to the participants. Instructors' role of guiding in a technology-enhanced instruction and collaborative environment provides students with learning opportunities and the necessity of exploration (Liang, 2016). According to this view and parallel with the second principle, the researcher adopted the role of a guide for the students in the collaborative classroom atmosphere. Additionally, the instructor administered a classroom instruction within a discussion environment where the students were given the opportunity to be aware of their confusions about the subject and what the correct outcome should be while working on their tasks about exponential and logarithmic functions. Throughout the course, the instructor guided the students first to work individually, then in groups, and at the end as a whole class to share their solution strategies and then reach a consensus on the correct solution of each task. In each task, the instructor moved around the class and asked questions to each participant about his approach in solving the questions in the task while they were working individually. When the participants moved on to the second stage where they worked in groups, the researcher observed their discussions and solution processes. In the last part where all class presented ideas about their solutions, the researcher guided the discussions by asking probing questions.

iv. Utilizing an educational technology.

McCulloch, Kenney, and Keene (2012) point out the importance of using graphing calculators in a mathematics course by asking the following question:

> *Imagine the following situation: You have worked out a problem by hand and decide to check your solution by doing the problem a second time using your graphing calculator. The solutions you get don't match. Which do you choose to trust? Why? (p. 464)*

In this study, the Desmos graphing calculator application was used as an educational tool to understand how students answer the variations of the above mentioned question in terms of calculation and graphing. Desmos graphing calculator was utilized as the main educational technology in the instruction of exponential and logarithmic functions booklet. Desmos is a technological tool that mainly allows writing equations of functions, draw the corresponding graph, and generate a table for the equation (Desmos, 2017). In addition to the basic functions, users can enter expressions with parameters. The expressions box automatically forms a slider for each parameter that you can adjust and explore the instant transformation on the graph. Basic menus such as save, add, undo, redo, zoom, and duplicate are easily accessible on the screen (Desmos, 2017). Moreover, the user can explore the transformation in graphs when equations of different types of functions were entered. The aims of use can be various according to the needs of the student. In this study, these features of Desmos were aimed to be utilized for completing the tasks in exponential and logarithmic functions instructional package. In the tasks that required writing algebraic expressions, drawing graphs, finding the domain and range of the functions, deciding the extremum points, finding x and y intercepts, and identifying the given function as increasing or decreasing, the participants were asked to seek for solutions first using pen and paper, then Desmos. Then, in the in-group discussions and classroom discussions they expressed how utilizing Desmos differentiated their solution, if it did.

Participants

The participants of the study included seventeen freshmen - 15 female and 2 male students - in the Middle School Mathematics Education Program at a private university in Turkey. The participants can be said to be homogenous in terms of educational background as all of them recently graduated from high school and enrolled in the Middle School Mathematics Education Program by receiving close

scores from the national university entrance exam. The age of the students was between eighteen and nineteen.

Convenience and purposive sampling methods were used to conduct this study. It was convenient to conduct the study with these participants because the first author of the study was the instructor of the Fundamentals of Mathematics course. Moreover, in this way, as a member of this class community, the researcher aimed to collect as much reliable data as possible through multiple data sources. In addition, the setting and the participants were appropriate for the aim of the study for two reasons. First of all, first academic year of their undergraduate education is the first time that the students are introduced exponential and logarithmic functions. In the following years, they work more on these functions in courses such as Analysis I, Analysis II, Analytical Geometry, and Differential Equations. The Fundamentals of Mathematics course was their first mathematics course in the university. It was considered worthwhile investigating and enhancing undergraduate students' practices before they construct further knowledge of exponential and logarithmic functions in other mathematics courses at the university. Secondly, the need for this research emerged from the actual needs of the students in the course. That is, the need for the study appeared in the first four weeks of the course through observation of students' practices in and understanding of functions concept. The students' preparedness was inadequate for constructing knowledge of specific types of functions, as they could not progress in defining and describing functions concept and types of functions. Prior to the implementation of exponential and logarithmic functions, pre-discussions were held with the participants to determine their deficiencies and misconceptions regarding functions concept in general and specifically exponential and logarithmic functions. Based on the instructor's classroom observation and discussions on functions, it was evident that the participants had limited idea on the algebraic representations and on graphical representations of exponential and logarithmic functions. Their earlier experiences with functions were limited to simple calculation skills to solve an equation of a function for a given value of x and they were actually unaware of a relationship between these two functions. Due to these reasons, the participants were perfectly suitable to the purpose of the study since they were in need of a support in learning exponential and logarithmic functions.

Data Collection Procedure

The data of this study were gathered through classroom observations, in class discussions, researcher's field notes, and reflection papers.

There was not a constructed observation protocol. Each participant's process of completing the tasks in the booklet was observed by the instructor of the course. Their written responses to questions of the booklet and/or conversations within the group of students constitute the content of the observations. The researcher who instructed the course, continuously observed and took notes about the tasks for which the participants chose to use Desmos, the features of Desmos they utilized, each participant's solutions before and after they utilized Desmos. Based on the classroom observation, the instructor asked the participants guiding questions to understand their utilization of Desmos while working on tasks about exponential and logarithmic functions. The leading views of each participant both within-group discussions and among groups were noted down simultaneously by instructor. To understand the participants' experiences from their own perspective, they were required to write about how they utilized Desmos while working on the tasks about exponential and logarithmic functions after they completed all the tasks in the instructional package.

The participants were assigned numbers from 1 to 17. The instructor's field notes were used to examine the features of Desmos graphing calculator that the students utilized in learning exponential and logarithmic functions. The focus of the analysis based on the sequence in the participants' solution processes while they were trying to solve the tasks via Desmos. Their actions while using Desmos were observed and such observations provided information about how and why they used graphics within the software, as well as what features of Desmos they applied to solve the problems in the tasks. In line with this, the analyses of these observations helped researchers to understand the reflections of the

participants while investigating their views about the support provided by Desmos while learning exponential and logarithmic functions. In these discussions, the connections that were essential to comprehend functions and exponential/logarithmic functions or to respond to the questions about exponential/logarithmic functions were clues for categorizing students' understanding of exponential/logarithmic functions.

The data obtained from multiple data sources were interpreted and discussed together by the two researchers. In the analysis process, the researchers separately coded the data. The results were compared, and it was seen that 95% of the codings overlapped. Discussions continued until reaching consensus on the 5% of diverse coding and then the analyses were finalized with total consensus.

Referring to what has been said in the related literature (Weber, 2002a; Weber, 2002b; Kenney & Kastberg, 2013; Williams, 2011; Hiebert & Lefevre, 1986), in this study, the students' own expressions and actions were examined in this context and were compared and contrasted with the findings of the previous studies in the literature. When any differences were observed, those were also interpreted in the light of the descriptions in the related literature.

FINDINGS

The findings of the study were presented and discussed with narratives of exemplary actions and supportive quotes related with the research question of *what are the views of undergraduate students about the use of Desmos graphing calculator in learning exponential and logarithmic functions?*

We aimed to answer to the research question by determining which features of Desmos were utilized to enhance students' progress in completing the tasks about and thereby their learning of exponential and logarithmic functions. The data obtained from classroom observations while participants were working on the tasks of exponential and logarithmic functions, from researcher's field notes, and from the related ongoing discussions as well as reflection papers revealed the reasons of the participants' specific preferences of Desmos over pen and paper and the advantages of using these features in learning the two function types.

It was found out that the participants utilized Desmos to view the expressions and the corresponding graphs of the functions on the same screen, displaying multiple graphs together on the same plane, marking every graph in different colors, identifying x and y intercepts of any point on the graph, graphing on an infinite coordinate axis with squared graphing background, drawing immediate and accurate graphs, zooming in and out, dragging, activating and deactivating the graphs, and utilizing the notations of functions that come ready within the menus of Desmos. The participants expressed that these features of Desmos altogether provided them with the opportunity to obtain accurate and immediate graphs and also they prevented the participants from making mistakes, inaccurate graphing, and getting confused. Moreover, the participants stated that the aforementioned features enabled them to observe the differences and similarities between the graphs of functions, to explore the relationship between algebraic and graphical forms of the functions, to compare multiple functions at a time, to identify the coordinates of all points on a graph, and to do more work in a shorter time. While working on the graphing tasks, participants firstly sketched graphs of the functions without using Desmos by giving values to variables or directly by guessing from the algebraic form of the function. In observations and discussions, however, it was observed that in all drawing tasks, all of the 17 participants corrected their initial graphs after comparing them with the graphs on Desmos. In this regard, this finding shows us that students use Desmos for making less error and control their solutions.

The analyses of the data obtained from classroom observation, in-class discussions, researcher's filed notes, and reflection papers revealed 3 main categories about the views of undergraduate students on the use of Desmos graphing calculator while learning exponential and logarithmic functions. These advantageous roles participants attributed to Desmos were i) compensating the lack of procedural knowledge, ii) providing opportunities for exploration, and iii) enhancing engagement with the tasks.

i) Compensating the Lack of Procedural Knowledge

The first category driven from the data analysis was the role of Desmos in compensating participants' inadequate procedural knowledge in terms of solving algebraic expressions of exponential and logarithmic functions. At the beginning of the study, it was observed that the students had deficiencies in procedural knowledge. For instance, they were uncertain about how to use the x and y variables, how to display them on the graph, the meaning of domain and range of a function, and different values of these functions. Moreover, although the students were familiar with the name of the exponential or logarithmic functions, they were not able to explain the relationship between those functions. In this regard, exponential functions and logarithmic functions were separate concepts in their minds.

The participants were able to link their existing procedural knowledge, which indicates that they knew how to assign values to x and make the calculation to find the corresponding value for y. However, they failed to plot the graph of the logarithmic functions as they were not able to correctly calculate the result of the logarithm for several values of x. They were better at calculating the values of exponential functions, still not obtaining successful results entirely. This failure in finding the correct result hindered them from completing the subsequent tasks such as analyzing the reason of increase or decrease of the function and explaining the change in the shape and position of a graph when the base of the function was changed. Accordingly, when the participants were observed while applying a corresponding approach, their procedural deficiencies also hinder their conceptual understanding by causing them to fail completing the tasks on pen and paper. At this point, the participants benefitted from Desmos. Despite the fact that the teacher did not direct the students to use Desmos, they discovered that they could compensate for their procedural deficiencies utilizing Desmos. It was seen that the participants (P1, P2, and P4) demonstrated progress in interpreting graphs with the support of Desmos. One of the participants (P1) stated their experiences as follows:

> *"I am not always good at solving exponential and logarithmic functions, especially logarithmic functions. Therefore, I could not pass on to the upper level of drawing their graphs before (in high school). But with Desmos, I was able to draw their graphs and answer all the questions about the related changes in algebraic form of functions and their graphs. The symmetry of two graphs was apparent on Desmos, $\log_3 x$ and $\log_{1/3} x$. It was easy to conclude that base of the function was effective on increasing or decreasing." (P1)*

As it is seen from the P1's comment, the participant created the graph via Desmos and was able to interpret the base and coefficient concepts of the logarithmic function on the graph displayed in Desmos. In line with this, although the student could not solve the questions about the functions, he/she thinks that it is important for him to understand the graph with the help of the program. According to the experience of the student, Desmos compensated lack of procedural knowledge of the student.

Before taking any Desmos support, three participants (P3, P5, and P13) drew almost accurate graphs of exponential functions by assigning values to both a and x. Although those graphs could be evaluated as correct, the slope and the position of the graphs were not exactly true. For instance, the graph of $f(x) = 3^x$ looked like $f(x) = 10^x$. The graphs of P3, P5, and P13 were correct considering the positions of the graphs according to each other. That is, the graph of the function having the largest positive value for base a was closest to x-axis. However, after entering the functions on Desmos, they corrected the positions and slopes of their graphs.

> *"I gave 2, 3, 4, 5, and 6 for the values of a. As these were close values, the graphs overlapped in my own drawing (on paper). Desmos helped me identify which graph is more close to x-axis. Moreover, I saw the curves of the graphs exactly and accurately. Now, I know how the graph of an exponential function looks like and how its shape changes when the base increases." (P13)*

As P13 mentioned, Desmos was helpful in interpreting the base concept of the logarithmic function and it was used as tool to check their graphs. Unlike the previous student, this student was able to draw his own graph at the beginning. The student identified and corrected the lack of knowledge about the axes

placement by determining the x values and y values through the program. In this context, although the student has the lack of knowledge in the process of drawing the graphs less than the previous student, the program helped the student to understand the interpretation of the bases of these functions on graphs. Although the deficiencies of the students' knowledge are at different levels, the program allows the students to recognize this lack of knowledge and to answer the questions despite the shortcomings.

This category emerged from the data showed that the students had lack of procedural knowledge for drawing accurate graphs, thus, used Desmos in order to create graphs and check their initial inaccurate graphs. Additionally, Desmos supported the participants to overcome the procedural calculation challenges, thereby explore exponential and logarithmic functions.

Ii) Providing Opportunities for Exploration

The participants' views were mostly gathered under the exploration category. In the classroom discussions, they indicated that they had the chance to explore and identify exponential and logarithmic functions in various aspects by using Desmos. These aspects were explained through selected narrative scripts from the courses.

In one group, the participants P7, P11, and P16 developed a strategy in which they traced over the graphs of two exponential functions with the bases of ⅗ and ¼ ($f(x)=3/5^x$ and $f(x)=1/4^x$) on Desmos and tabulated the x and y values. They simply pointed on the graph tracing to the integer alues of x; the corresponding y coordinate of the point appeared on the graph immediately, and they tabulated these values on a paper. With this strategy, they aimed to compare the values of x and y and identify the relationship between the algebraic expressions of the two functions. Before using Desmos, they only sketched the graphs for positive x values, which appeared only in the first quadrant. When they used Desmos, the graphs appeared as a whole in first and second quadrants which enabled them to explore the graphs for both positive and negative values of x. At the end of the task, P11 described her exploration process of graphing on Desmos as follows:

> *"If I were to draw these two functions in the way we used to (assign values to x and calculate corresponding y values), I would only draw the graphs on the first quadrant of the coordinate plane and relate the functions relying on that part of the graph. However, on Desmos, I was able to explore all parts of the graph. I observed that the graphs changed position when they continued to the second quadrant of the coordinate plane. Therefore, we (group members) decided to add x and y values from second quadrant, too. After we tabulated the coordinates from both quadrants, we were able to interpret the relationship between the change in the base of the exponential function and the shape of the graph." (P11)*

Considering P11's explanation, it was evident that Desmos enabled her to examine the x and y values exactly with the grid paper feature and comprehend the graph as a whole in both quadrants of the coordinate plane. Similar to P11, another participant (P7) from the same group indicated that she was able to explore the left part of the graph corresponding to the decreasing negative values of x.

> *"By dragging and zooming, we were able to move to the points we wanted to and found y values for any x. You can even see the end of the graph." (P7)*

In this regard, all parts of the graph were explored through the dragging and zooming features of Desmos which also made it possible to reach any points on the infinite graphs they came up with at the end of the task.

The booklet began with a task about writing the algebraic expression of an exponential function for a given situation. Then, the first task continued with the questions about calculating the values in the exponential function; x or y. Following this, in the second task, the participants were asked to calculate the $f(x)=y$ values of exponential functions with the bases of 2, 3, and 5; $f(x)=2^x$, $g(x)=3^x$, and $h(x)=5^x$. While doing this, they assigned different values to x. Then, they used these x values to find y values and sketch the three graphs. The participants compared their graphs with each other and discussed the differences. Afterwards, they used Desmos to graph the three functions. The comparison questions at the end of the

task were "write the set of the *x* values where *f(x)>g(x)*", "write the set of the *x* values where *f(x)<g(x)*", and "write the set of the *x* values where *f(x)=g(x).*" They observed the graphs on Desmos and easily found the sets of *x* values - the domain sets - for the given situations. P9, for instance, stated that she did not need to calculate values for *f(x)*, *g(x)*, and *h(x)* functions to answer the questions in the task because on Desmos she was able to explore the conditions given in the questions.

> *"We don't need to make any calculations or sketching. We can just enter any exponential functions to Desmos with bases like 5 and 15. Then, we can easily view the sets for x when these functions get higher values from each other (f(x)>g(x) and f(x)<g(x)) or when they are equal (f(x)=g(x)). It is apparent on the graph on Desmos." (P9)*

In a similar task in the logarithmic functions, the participants skipped the beginning steps and directly entered the algebraic form of the functions to Desmos; $y=log_2 x$ and $y= log_5 x$. Then, all of them were able to find the sets for *x* values for given situations by exploring the graphs on Desmos. Upon completing the task, they were asked to compare the graphs by referring to all of the differences and similarities. P8's view was remarkable in that she suggested the use of Desmos to make this comparison thoroughly rather than using pen and paper.

> *"On Desmos, you can see that as the base of the logarithmic function increases, the graph moves closer to x and y axis. I mean, positive x and negative y axis. I couldn't recognize this on my own graph (on the paper). On Desmos, I entered functions with consecutive numbers and the appearing shapes are explicit enough to compare and the overall shape is impressive. Look (showing the graphs on Desmos screen on her mobile phone)."(P8)*

P8 supported her idea by presenting the graphs in Figure 1 on her mobile phone.
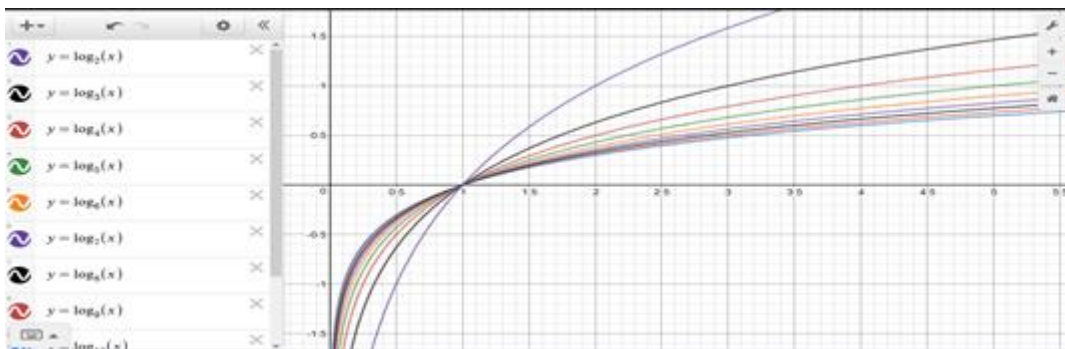

Figure 1. Desmos screen provided by P8

In the tasks requiring comparisons of graphs, the participants put an emphasis on the accurate and precise graphing. Hence, they used Desmos to write explicit answers to these comparison questions. In addition to their statements, it was evident in the classroom observation that the participants could not provide complete answers to the question of "*what is the difference between the rate of increase/ decrease of the logarithmic graphs when 0<a<1?*" In the task, the value for the base of the logarithm was not specified. Instead, participants worked in groups and every group determined their own strategy. Although everybody chose a smart value for base *a* which was ½ and assigned computable values to *x* such as ½ and 1/16, there were no accurate drawings on paper and no true comments were provided for this question. The groups discussed the question as a whole class by referring to the most accurate drawing on the paper drawn by P2 and P12. Some of the partial answers were as follows.

> *"The graph moves towards x-axis for positive values of x." (P4)*

> *"For positive (value) x, for example 9, it (the graph) decreases." (P15)*

As it was evident in this observation script, the deficiency in drawing hindered the participants from finding answers to the questions requiring higher order thinking skills. Then, participants were allowed to use Desmos. They entered $y=log_{1/2} x$ to Desmos. They all made the right comment that graph cannot be defined for negative values of x and $x=0$. Then, they entered other values for base a between 0 and 1 to make comparisons between the graphs. P9 expressed her group's result by showing her Desmos graph (Figure 2) as follows.

> "We assigned two values for the base; ½ and 7/10. The graphs appeared in first and fourth quadrant of the plane. It is evident in the graph; it decreases. So, we can conclude that logarithmic functions with bases between 0 and 1 are decreasing functions. It is also evident that as the base increases, the rate of decrease also increases. Look, the graph of $y= log_{7/10} x$ decreases with a higher slope." (P9)
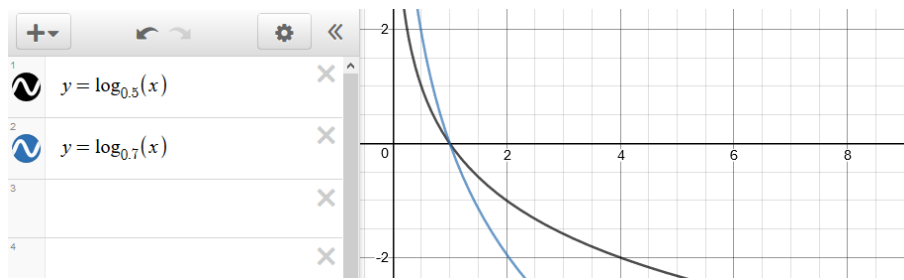

Figure 2. Desmos screen provided by P9

Then, in the following questions where domain sets of the logarithmic functions for $0<a<1$ and $a>1$ were asked all of the participants provided complete answers by adding $y=log_2 x$ to the previous graphs on Desmos. Although it was not asked, all of the participants recognized the symmetry between the logarithmic graphs with reciprocal bases such as 2 and ½. All participants stated that $y=0$ is the symmetry axis by showing their graphs on Desmos as the evidence for this claim.

The first section of the booklet was on exponential functions. The second section started with the relationship between exponential and logarithmic functions. The inverse relationship between the functions were instructed by changing places of x and y variables in exponential functions and trying to define y. By making a list of calculations for given x and y values, the participants got used to the algebraic expression of logarithmic function for its related exponential function. Then, when they entered functions with varying bases on Desmos, they immediately recognized the inverse relationship. P14 expressed the symmetric graphs and relationship between x and y values as follows.

> "I entered $y=log_5 x$ and $y=5^x$. On Desmos when you look at the graphs, you can see that they are symmetrical according to $y=x$ line. It is also apparent that domain and range sets interchange. It makes sense because they are inverse functions where x and y change places." (P14)

At the end of a brief discussion, upon observing their graphs on Desmos, all participants recognized the inverse relationship regarding the location of the graphs and the domain and range sets of an exponential function and its related logarithmic function. All of the participants also expressed that coloring feature enhanced their exploration.

> "It allows graphing multiple functions on the same coordinate axis with different colors, so I could compare the change on the graph when I change the base of the logarithmic function." (P10)

Iii) Enhancing Engagement with The Tasks

All participants actively involved in the courses by working on the tasks about exponential and logarithmic functions. That is, all of them developed strategies to complete the tasks, solved the

functions for given values, wrote algebraic expressions of functions, drew graphs, identified the domain and range sets by referring at the graphs, compared the given functions, searched reasons for the increase or decrease of functions, identified similarities and differences between functions, and related algebraic and graphical representations of functions. Although they did not succeed in all steps, all participants responded to all questions in the tasks by utilizing Desmos and all of them were willing to express their answers in class.

In addition to the participants' statements in the discussions, it was also evident in the classroom observation that the support provided by Desmos motivated and encouraged all participants to engage in the tasks including those having low mathematics success. For instance, P9 expressed that she felt comfortable in attending the exponential and logarithmic functions tasks. The reason was that Desmos provided immediate and accurate feedback.

> *"Desmos was a helper which always knows the truth. This felt very comfortable, so I was enthusiastic to complete the tasks with everyone and I tried to catch others in the class. Without Desmos I would not even initiate to answer any of the questions. I would wait until you write the correct answer, so I can write it down, too." (P9)*

Depending on similar views of participants, the use of this software, supported by group work, supports the motivation for solving the questions. As the participants pointed out in their comments, they felt confident that they can complete tasks with Desmos. However, that feeling reveals the over dependency of students about using Desmos for solving each task. If the student can not answer any questions without Desmos, it is the result of the trust of students in Desmos. As a positive follow-up impact of utilizing Desmos to the previous impact (i.e., compensating the lack of procedural knowledge), Desmos helped the participants compensate for the lack of content knowledge; therefore, it increased their self-confidence together with their amount of engagement with the tasks.

Similarly, P5 and P1 were encouraged to draw graphs since Desmos helped them about drawing accurate graphs and provided multiple graphs to compare the positions and behaviors of the graphs.

> *"I don't like to draw the graphs of a list of functions because it takes too much time. Even if I give that time and effort to solve the function and find the coordinates of the graph, the drawing will not be accurate. Then, I compare the graphs and make wrong or missing comments. Using Desmos saves time and brings out true graphs. This enabled me to compare a list of exponential and logarithmic functions with various bases." (P5)*

Moreover, it was observed that using Desmos enabled the participants to speak up without any fear of making mistakes and they used Desmos as reference to their answers. Another student, P14 indicated that he used Desmos to check his answers and relying on Desmos, he was encouraged to make comparisons between logarithmic functions with different bases.

> *"I recommend everyone (undergraduate students) to use Desmos. It is useful to check your answers. Sometimes it is complicated to calculate y for a given x in logarithmic functions. It is also hard to draw the graph of these functions such as drawing with an accurate slope and position. To check my answers, it was really a good opportunity to use Desmos. When I saw the graphs of the logarithmic functions with different bases on Desmos, I knew I was making true comments on the behavior of the graphs according to their bases." (P14)*

In accordance with P14's comment above and the previous one, Desmos was a tool that allowed participants to reach correct interpretations. Likewise, she benefited the program to check her solutions. In addition to this, drawing functions with different bases was considered easy with the support of Desmos. Therefore, she benefited from the program to provide an answer as well as to check her answer that she found without Desmos.

The freshmen who had different backgrounds and levels in computer skills adapted themselves to using Desmos easily. They considered the mobile application of Desmos as a user-friendly program. P2 expressed that Desmos was a user-friendly tool enabling numerous function entries at the same time. Thus, she was eager to draw graphs and answer the questions in the tasks.

> *"I entered a lot of versions, even the ones you did not mention. I changed the coefficients and observed the changes in graphs. It was like trial and error game. It does not take too much time like drawing on paper. It is easier to answer the questions this way." (P2)*

We see that the program supported P2 with exploration of changes in various graphs. The participant drew the functions on Desmos as well as examined the change in the graphs.

DISCUSSION & CONCLUSION

Fundamentals of Mathematics course in Middle School Mathematics Education Program aims to develop undergraduate students' knowledge in basic mathematics subjects. It provides mathematical basis for the higher-level courses in the following grades such as analysis, linear algebra, analytical geometry, and differential equations. Moreover, undergraduate students take two methodology courses in the third grade. In these methodology courses, in order to design courses and provide suggestions to teach mathematics in the middle school, the prospective teachers need to be competent in mathematics content knowledge. Hence, the course provides bases for profound understanding in mathematics for undergraduates who will be middle school mathematics teachers in the future. However, this course covers a challenging content for freshmen who lack mathematical knowledge and skills. Based on the researchers' teaching experiences and the related literature, undergraduate students have inadequate and inaccurate understanding of exponential and logarithmic functions (Hurwitz, 1999; Kenney & Katsberg, 2013; Weber, 2002a). Therefore, in the present study, technology utilization in mathematics education was used to improve undergraduate students' learning of exponential and logarithmic functions. In this regard, in this study the undergraduate students' views about and actions in the use of Desmos graphing calculator in learning exponential and logarithmic functions were investigated.

Desmos is described as a user-friendly tool (Oates, Sheryn, & Thomas, 2014; Thomas, 2015). Similarly, all of the participants in this study expressed that Desmos was user-friendly and they were used to all its menus and its usage at the end the very first course. They did not ask any questions about how to operate a command on Desmos after the first course.

Participants mentioned in classroom discussions and reflection papers, the features of Desmos that substituted pen and paper were related to dynamicity, visualization, motivation, and user-friendly aspects of the software. A similar result was obtained in the study by Karadeniz and Thompson (2017) in which they stated dynamic environment of graphing calculators and visualization enabled students to explore mathematics. In the reflection papers, it was evident that these features offered learners an effective setting for drawing related graphs, but cautiously interpreted for learning mathematics. Because, in some cases (e.g.: drawing multiple graphs for interpreting the behavior of the graphs of functions with differing bases), participants mentioned about Desmos usage in shortening the process of responding to the question, in drawing the graph in a limited time, and in using trial-and-error methods. That made the researchers think that students might have used the program as a means to give the desired answer to the question, rather than learning mathematics. Moreover, the use of trial and error method needs to be carefully evaluated because the strategies developed during this method are also important. What is done by trial and the paths students use are required to be carefully examined. Otherwise, effective learning might not be acquired. The performance of the calculator may hide and hinder mathematical skills and students' progress can be artificial (Zheng, 1998). In addition to this, *taking too much time* is often a concern expressed by the participants. As opposed to what the participants say, the essential skill during the learning should not be drawing a graph immediately. Rather, it is important to draw graphs manually by paying attention to the steps of forming them.

The first inference from the findings of the study was that the participants compensated their lack of procedural skills by engaging in the learning process through the use of Desmos. Even though they had difficulty in solving exponential and logarithmic functions, by using Desmos they were able to graph and interpret the functions without being challenged by calculation. Technology has the power to move the focus from computation to conceptual understanding (Esty, 2000). In standards of National Council of Teachers of Mathematics (2000) it is emphasized that technological tools "furnish visual images of mathematical ideas, facilitate organizing and analyzing data, and can compute quickly, efficiently, and accurately so that students can focus on conceptual understanding" (p. 24). In line with this finding, students may prefer this software because it is easier and faster to do their own calculations. In other words, the use of this software may have led them to underestimate their insufficient procedural skills, rather than progressing to the desired level in the computing skills of the students. Graphing skills lie in the base of learning functions. However, computation is also a desired skill that the students need to develop.

The second significant finding of the study was that the participants considered Desmos as an assistive tool which provided exploration. Throughout the process of working on exponential and logarithmic functions tasks, the participants were able to explore the graphs as a whole, observe all points on a graph and identify their coordinates, compare the positions and behavior of multiple graphs at a time, explore the relationship between algebraic and graphical forms of functions, recognize the inverse relationship between exponential and logarithmic functions, and define and interpret this relationship on graphical and algebraic representations through Desmos. It should also be noted that, the participants emphasized the advantages of using Desmos especially for the reason that Desmos enabled them to learn what they would not achieve through pen and paper. First and foremost, they emphasized comparing functions from several aspects such as algebraic form of functions according to graphs, domain and range sets, and exploring all points on a graph. Among the aspects the participants mentioned, relating algebraic form of functions with the corresponding graphs was an important outcome which was also stated in previous studies. In terms of symbols and notations in the algebraic form, it was noted that students have difficulty in thinking logarithm as a function (Hurwitz, 1999). Moreover, King (2017) found out that Desmos aided in exploring domain and range of functions and change in the graphs of functions according to the change in the algebra formula of the functions. The above mentioned two roles attributed to Desmos - i) compensating lack of procedural knowledge and ii) providing opportunities for exploration - were advantageous for the freshmen to analyze, understand, associate, and interpret exponential and logarithmic functions. Correspondingly, it can be cautiously concluded that utilizing Desmos graphing calculator in learning exponential and logarithmic functions facilitated undergraduate students' conceptual understanding by providing accurate graphical representations and enabling detailed exploration. Similar results were found in Quesada and Maxwell's (1994) study in which graphing calculators provided a learning environment that fostered students' construction of knowledge by its graphing utility in precalculus course.

Moreover, Bourassa (2014) suggested the use of technology in order to use both in solving and checking the solutions in functions problems. Parallel reasons for calculator use emerged in this study because the participants benefitted from Desmos in controlling their answers, making fewer mistakes, and thereby thinking relationally of those graphs. As well as the technological infrastructure that the software supports, those features also reveal the variety of experiences students wonder while creating graphs of those functions that cannot be done easily with pen and paper. It is interesting to note that the participants never insisted on telling that their graphs were correct. Instead, they tried to figure out how the graphs of those functions can be drawn using the properties mentioned above. Therefore, this program has the mathematical authority within the classroom.

As Weber (2002b) and William (2011) identified, students need to learn exponential and logarithmic functions as processes. The real-life tasks in the instructional package and the use of Desmos while working on those tasks provided learners with meaningful understanding. Desmos facilitates meaningful learning of functions (Karadeniz & Thompson, 2017). If it is used as an educational

technology and integrated with appropriate pedagogical and assessment strategies, it has the potential to enhance and improve conceptual understanding (Thomas, 2015). In this study, the results showed that Desmos allowed the participants to explore two types of functions deeply. Furthermore, they discovered the relationship between these two functions. Recognizing and identifying the inverse relationship between exponential and logarithmic functions is a complicated level of understanding (Gramble, 2005). As Kenney and Kastberg (2013) stated students need to build the relationship by considering logarithm both as a function and as an exponent. In this study, it was evident in the classroom observations and in their written answers to the tasks that through the use of Desmos, all participants reached the intended goals of the three-week long implementation about exponential and logarithmic functions. The gains obtained by the support of Desmos were infeasible through traditional teaching approaches.

The third main support the participants mentioned was that Desmos enhanced engagement with the tasks. It was observed in the class hours that they were eager to solve the problems about exponential and logarithmic functions by using Desmos. Moreover, they felt comfortable to express their thoughts and related their answers to the output in Desmos. They attributed their engagement with the tasks to Desmos because it was there to support or correct them while working on the tasks. All participants expressed that Desmos was advantageous in their learning because it provided immediate and accurate feedback, acted as a tool to check answers.

RECOMMENDATIONS

In this study, the use of Desmos as an educational tool aimed at facilitating undergraduate students' learning of exponential and logarithmic in undergraduate education at the faculty of education. The use of Desmos graphing calculator in teaching exponential and logarithmic functions also extended beyond being a substitute of pen and paper. Considering these results of this study, it has the potential to support learning in functions concept. It would be efficient to consider the support of Desmos in a true experimental design that supposedly would provide sharper results when compared to a controlled group.

It is necessary to express functions with multiple representations and Desmos provides exploration for algebraic, tabular, and graphical representation types. In this context, this application can be integrated into the undergraduate mathematics courses for students' meaningful learning. Therefore, it can be suggested that Desmos is a suitable tool for students to learn and construct functions throughout mathematics courses. On the basis of the participants' experience, it is necessary to clarify in what extend students are expected to have procedural skills about those functions and whether students have sufficient procedural knowledge when they answer questions via Desmos. Although the operational skills of the participants were not the only criterion in this study, development of procedural skills regarding functions can be investigated in a graphing calculator supported environment.

McCulloch et al. (2012) stated that calculus students' preference to use a graphing calculator has the following rationales in their learning process: 'concerns about making careless errors', 'the need to check work (their own written work or the calculator's work) before making a choice', 'beliefs about graphing calculator affordances and limitations', and 'confidence in their own mathematical ability' (p. 465). In contrast to the findings of the study mentioned, in this study participants did not share ideas about 'beliefs about graphing calculator affordances and limitations', and 'confidence in their own mathematical ability'. In this study, the participants expressed that graphical calculator was as a highly competent tool, yet they did not mention any possible limitations of the graphic calculator. Additionally, in this study participants did not trust their mathematical knowledge. Therefore, students who are confident in their mathematics knowledge can use Desmos in different ways.

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