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PROSPECTIVE TEACHERS' KNOWLEDGE OF CONNECTIONS AMONG EXTERNAL REPRESENTATIONS IN THE CONTEXT OF PROPORTIONALITY

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Abstract

Mathematics not only consists of procedures, symbols or operations, but also involves connections, representations, problem solving, reasoning and proof, communication, and conceptual understanding. Learners should make necessary connections among mathematical concepts and representations in order to develop deep mathematical understandings. For this reason, this study aims to explore prospective middle school mathematics teachers' views about mathematical connections and their knowledge of connections among external representations in the context of proportionality. Multiple case study was used as a research design and the participants were three prospective middle school teachers enrolled in a teacher education program at a public university in the inner region of Turkey. Data were collected through semi-structured interviews, note taking, and written tasks. The findings of the study suggested that prospective teachers viewed connections mainly as a link between different mathematical topics or concepts, as a link between mathematics and daily life, and as a tool for improving students' understanding of mathematics. Moreover, prospective teachers were able to translate easily among different representations. However, the translations were carried out without conceptual understanding. Besides, participants had limited understanding of proportionality in the case of graphical representations.

Keywords: Prospective teachers; mathematical connections; external representations; proportionality

INTRODUCTION

In today's world, the importance of knowledge is increasing rapidly, as a result, "knowledge" concept is changing and therefore this is leading to the need for change in individual skills. A shift is needed in every area of life including education (Ministry of National Education [MoNE], 2017). For at least two decades, mathematics education has undergone slow but steady changes by public or political pressure. The Common Core State Standards for Mathematics (CCSSM) is an important document that continues to guide a radical reform movement in mathematics education not just in the USA but also in many countries. The recent curriculum documents in mathematics education demand less emphasis on computation and knowledge of algorithms and a greater emphasis on connections, representations, problem solving, reasoning and proof, communication, and conceptual understanding (e.g., CCSSI, 2010; MoNE, 2017).

Mathematical connections are components of school mathematics curriculum goals (National Council of Teachers of Mathematics [NCTM], 1989). Making connections is among the most challenging ideas

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for students to achieve. However, it is especially very important at the middle school level where students first begin to recognize the real value of mathematics (Reed, 1995). Mathematical connections may be used to link between different mathematical ideas, to link mathematical ideas to other disciplines, and to link mathematical ideas to students' daily lives. These connections help students develop a more robust understanding of mathematics and perceive it as a beneficial and an exciting subject to work on (Reed, 1995). According to Ma (2010), understanding can be explained in terms of making mathematical connections. She defines "profound understanding" of mathematics as the ability to connect ideas within a topic and to central concepts of the discipline. This shows some evidence that mathematical connections are closely related to the concept of understanding. Similarly, Baykul (2009) states that conceptual knowledge is a set of mathematical concepts that are connected to each other. Thus, mathematical connections may help to develop conceptual understanding.

Mathematical connections are among five process standards and refer to the process of mathematics through which students gain and employ mathematical knowledge. This process standard is summarized by the NCTM as follows:

> "Instructional programs from prekindergarten through grade 12 should enable all students to recognize and use connections among mathematical ideas, understand how mathematical ideas interconnect and build on one another to produce a coherent role, and recognize and apply mathematics in contexts outside of mathematics" (p. 64).

The statements mentioned above emphasize that mathematical connections are inevitably used to bridge between various mathematical ideas, between mathematics and the real life, and between mathematics and other disciplines. Moreover, mathematical connections have been one of the most crucial targets of Turkish middle school mathematics curriculum. According to MoNE (2017) mathematics not only includes procedures, symbols, figures, and operations, but also consists of a web of meaningful connections. Hence, Turkish middle school mathematics curriculum expects students to benefit from and make connections within mathematics, make connections between mathematics and other disciplines, and with real life situations, connect different representations of concepts, procedures, and contexts, make connections among different representations, be self-confident when making connections, and display positive attitudes towards making connections.

Davis, Young, and McLoughlin (1982) define a representation as "a combination of something written on paper, something existing in the form of physical objects and a carefully constructed arrangement of ideas in one's mind" (p. 54). Hiebert and Carpenter (1992) state that the development of understanding partially depends on the ability to construct connections within and between representations. They classify representations into two as internal and external representations. Symbols and physical objects are considered to be external representations and mental images are considered to be internal representations. They suggest that internal and external representations play a crucial role in the development of understanding for several reasons. Namely, they emphasize that internal representations are used while reasoning about mathematics, they are converted to external representations while communicating mathematics, and the construction of connections among them are simulated by building connections among external representations.

The current study concerns with students' external representations. Therefore, symbolic, tabular, and graphical representations need to be highlighted. By the help of external representations, we can easily communicate to other people (Coulombe & Berenson, 2001). External representations can be represented physically and they are visual configurations such as equations, tables, and graphs (Goldin & Kaput, 1996). Verbal representations (e.g., written words), graphical representations (e.g., cartesian graphs), algebraic or symbolic representations (e.g., equations), pictorial representations (e.g., diagrams or drawing), and tabular representations are among the most common external representations. Many mathematical ideas such as multiplication, geometric problem solving, and diagrammatic reasoning involve the use of external representations (Zhang, 1997).

Lesh, Post, and Behr (1987) point out that translating among different representations (e.g., symbolic, tabular, and graphical) is a robust indicator of conceptual understanding. Hiebert and Carpenter (1992), Kaput (1989) and Porzio (1999) state that using multiple representations such as verbal, symbolic, tabular, and graphical representations to make connections among mathematical relationships and mathematical problem situations enriches students' understanding of mathematical concepts and relationships. Similarly, Driscoll (1999) argues that mastery in making connections among multiple representations is essential for the growth of students' mathematical understanding. The NCTM (1989) emphasizes that representations are one of the main elements of school mathematics curricula and more attention should be paid to them during mathematical instruction. Besides, it suggests that students who are able to translate within and among multiple representations of the same mathematical idea may solve mathematical problems more flexibly and develop an increased awareness about the coherence and elegance of mathematics.

This study focused on exploring prospective teachers' connections in the context of proportionality. Adjiage and Pluvinage (2007) define proportionality as a linear relationship between two variable quantities. In this study, there are two requirements for a relationship between two variables to be proportional: linearity and passing through the origin. Thus, in this study, proportional situations refer to special cases of linear situations passing through the origin. Lamon (2006) explained that proportionality is far more comprehensive than proportional reasoning. She further stated that proportionality has a role in applications governed by physical rules whereas proportional reasoning is a measure of learners' comprehension of basic mathematical ideas and it forms the basis of more complex mathematical concepts. Nevertheless, proportional reasoning is a precondition for understanding contexts and applications based on proportionality. Accordingly, mathematics educators use proportionality and proportional reasoning concepts interchangeably.

The CCSSM considers proportional reasoning as an important part of mathematics and it expects students to examine proportional relationships and utilize these relationships when solving mathematical and real-world and problems (CCSSI, 2010). In a similar fashion, Umay (2003) indicates that proportional reasoning forms a basis for all mathematical procedures and operations. According to Lamon (2006), proportional reasoning refers to "detecting, expressing, analyzing, explaining, and providing evidence in support of assertions about proportional relationships" and in everyday language it refers to "reasoning up and down in situations in which there exists an invariant (constant) relationship between two quantities that are linked and varying together" (p. 3). Therefore, proportional reasoning is essential for students to understand mathematics. According to the NCTM (1989), proportional reasoning ability develops in students as they progress through grades 5 to 8 and it is worth spending a great deal of time and effort in order to ensure that the students have carefully developed it. Such reasoning is crucial for solving daily life problems and for grasping not only advanced mathematical ideas but also other subjects such as physics and chemistry (Post, Behr & Lesh, 1988). Ratios and proportions are commonly used in mathematics, science, and real- life (Karplus, Pulos & Stage, 1983). These concepts act a bridge between the numerical, concrete mathematics of arithmetic and the abstraction that occurs in algebra and higher mathematics (e.g., Fuson & Abrahamson, 2005; Lamon, 2007; Post et al., 1988). Therefore, ratio and proportion concepts comprise critical thinking skills and are associated with many other mathematical concepts (Akkuş & Duatepe-Paksu, 2006).

Given the emphasis of national and international curriculum documents on connections, it needs to be explored to what extent educational organizations such as schools or universities give the opportunity to students, prospective teachers, and in-service teachers to make mathematical connections. Besides, proportionality is crucial for learners since it is a core concept and functions as "the capstone of children's elementary school arithmetic and the cornerstone of all that is to follow" (Post et al., 1988, p. 93-94). Therefore, the purpose of this study is to explore prospective middle school teachers' connections among external representations in the context of proportionality. For this purpose, answers to the following research questions have been sought:

1. What are prospective middle school mathematics teachers' views about mathematical connections?

2. How do prospective middle school mathematics teachers make connections among external representations in the context of proportionality?

RELATED STUDIES

There is a scarcity of research on the nature of connections learners construct in mathematics (Adu-Gyamfi & Bossé & Chandler, 2017). However, research related to connections gathered mainly around two themes as real life connections with mathematics (e.g., Gainsburg, 2008; Lee, 2012) and connections within mathematics (e.g., Adu-Gyamfi et al., 2017; Debock, Neyens & Van Dooren, 2017; Eli, Mohr-Schroeder & Lee, 2011; Orrill & Kittleson, 2015). Related studies are reviewed as follows:

Using a phenomenological research design, Gainsburg (2008) surveyed, observed, and interviewed secondary school mathematics teachers and explored their comprehension and utilization of real life connections, their purpose for making connections in the course of teaching, and factors that promote or hinder their use of connections in the teaching of mathematics. Gainsburg found out that teachers mainly used connections when presenting word problems and planned examples, or when students were working on a project. The teachers added some more properties to connections to improve the authenticity of connections such as using real artifacts as reference, relating examples to students' personal situations, and using real data generated by students. Teachers mainly connected mathematics to the following real-world contexts: structural or interior design, shopping/pricing/eating out, banking/budgeting, and cars or other means of transport. Teachers reported that most of them made real life connections at least weekly or monthly. Teachers indicated that they mainly used their own ideas or their past experiences when making connections. Some other sources they used for making real-world connections were course textbooks, professional development workshops and colleagues. Teachers indicated that they mainly used connections for the purpose of motivating students, drawing students' attention to the topic, making concepts easier for students to understand, having student recognize that mathematics is all over the world, having students believe that mathematics is used in daily lives. Teachers also reported that they could not use connections in their lessons due to the following reasons: connections are time consuming, they require some training to be able to use effectively, and they are not emphasized in the curriculum. Gainsburg suggested that teacher educators should design professional development activities to train teachers about how to make connections, to show that connections strengthen mathematical mastery, and to help teachers understand what turns mathematical tasks into real-world tasks.

Lee (2012) conducted a case study to explore prospective elementary teachers' perceptions of real life connections related to word problems. The participants were asked to complete a two-part assignment. In the first part, they were asked to collect and create two word problems that they think best represent real world connections. In the second part, the researcher selected 10 of the word problems and asked the participants to rate these word problems on a 5-point scale (5 referred to the highest quality of real world connections). Lee found out that participants strongly believed that real life connected word problems should certainly be used in mathematics education. However, the points they considered about connections was not enough for the researcher to determine the extent of their knowledge about the characteristics of connecting mathematics with real life. The prospective teachers perceived that usefulness and reality were essential parts of real life connections. Besides, there were huge inconsistencies between participants' beliefs and the way they created or evaluated word problems in terms of real world connections. Lee concluded that there was a gap between what prospective teachers thought and what they did. She further suggested that participants were aware of the importance of using real-world connections in general, however they could not think as well as in-service teachers about the implementation of real life contexts in the teaching and learning of mathematics.

Adu-Gyamfi et al. (2017) conducted a qualitative study to explore the nature and misconceptions of six high school students' connections among algebraic and graphical representations in the context of a polynomial relation. The researchers observed and interviewed the participants while they were performing a non-standard polynomial task (i.e., different from student textbook tasks). This task measured participants' mathematical competencies through three fundamental concepts as reversibility, flexibility, and generalizability. The participants worked in cooperation in two groups. The researchers proposed that the nature of participants' interpretation and interplay with representations were contingent upon their knowledge of representation system, knowledge of domain, and knowledge of domain register. The results of the study showed that exposing students to reverse thinking, flexibility in using approaches, making generalizations through specific examples, and extending thinking about polynomial functions uncovered limitations in participants' aforementioned knowledge types. The researchers noticed that standard instructional practices which do not remedy students' lack in the aforementioned knowledge types would do little in understanding the larger picture of representations. They suggested pre-calculus instructors to use tasks that force reversibility, flexibility, and generalizability in order to cover functions more effectively and to bring to light students' misconceptions while they are making connections among representations.

Eli et al. (2011) used a mixed methods design to examine prospective middle-grades teachers' ability to make connections while engaging in card-sorting tasks. The card-sorting activity was designed in a way that enabled the researchers to investigate the type of mathematical connections the participants made among different mathematical concepts, definitions, and problems. The analysis of data revealed that participants used five different types of mathematical connections as categorical, procedural, characteristics/property, derivational, and curricular. However, the majority of the connections made by the participants were categorical and procedural and there were much fewer derivational and curricular connections. The researchers attributed the predominance of categorical and procedural connections to the fact that the participants followed a traditional curriculum that emphasized procedural rather than conceptual understanding of mathematics and that the participants had still not taken mathematics methods courses. The researchers concluded that the types of mathematical connections emerged in their study may help those who want to create mathematical connection tasks for the purpose of empowering prospective teachers' conceptual understanding of mathematical concepts.

Orrill and Kittleson (2015) conducted a case study to explore how a middle school mathematics teacher might use connections in her own classroom after being engaged in a professional development program (PD). More specifically, the researchers investigated the connection-making moves experienced by the teacher during PD and in her actual classroom practices. The researchers developed a framework to analyze the teacher's connection making in both settings. Besides, the teacher was administered a questionnaire involving rational number items as a pre-test and post-test for the assessment of her growth of content knowledge after engagement in the PD. The framework concentrated on four aspects as "promoting mathematical explanation, promoting representation use, embracing multiple approaches, and scaffolding for cognitive demand" (p. 276). The duration of the PD was 14 weeks (42 hours) and during this period, the teacher was provided the opportunity to engage in explorations regarding fraction multiplication, fraction division, and proportional reasoning. The teacher implemented in her classroom a task that she had experienced in the PD and this allowed the researchers to examine how she translated her experience to the classroom. This task included two sub-tasks which were related to direct and indirect proportions. It was developed by the researchers and was more challenging than typical textbook tasks to support teacher learning. The researchers found out that the teacher's engagement in the PD was not sufficient for her to translate connection making moves into her teaching and they pointed to the challenge in influencing classroom practices through professional development programs. The researchers also noted that the teacher went on to use the simple up/up and up/down terminology for direct and indirect proportions, respectively after engaging in the PD. The researchers noted that the teacher's content knowledge grew considerably between the pre-test and post-test and however that she could not reflect this improvement to her classroom practices. The researchers suggested that there might be two different knowledge as "knowledge of content for oneself" and "knowledge of content for teaching" and argued that "the teacher may have abundant knowledge resources, but something about the context of the classroom invokes only a particular subset of those resources" (p. 294).

Unlike the aforementioned researchers, Debock et al. (2017) conducted a quantitative study to investigate high school students' correctness in connecting functions to their corresponding properties and determined whether this correctness was dependent upon function types (proportional functions, inverse proportional functions, affine functions with positive slope, and affine functions with negative slope) and representations (formulaic, tabular, and graphical modes). The students were randomly divided into three experimental groups and each group received the aforementioned four types of elementary function tasks in one representational mode. The results of the study revealed that the students displayed quite high performance in connecting functions to their own properties. This suggested that the students had sound concept images about elementary functions and their properties. The researchers also found out that students displayed the lowest performance in inverse proportional functions. Representational modes of tasks influenced students' accuracy rates in making connections as well. It was found that while tabular representations were more supportive for students than formulaic and graphical ones and the difference between the formulaic and graphical representations were non-significant. The researchers noted that their studies had some limitations owing to its nature and design. Namely, they indicated that collection of qualitative data might have uncovered students' strategies in making connections between functions and their properties. Second, they argued that the generalizability of their findings were limited due to the fact that the task administered to the participants was artificial and was completely different from the ones confronted in the classroom.

Although proportional reasoning plays a crucial role in learners' understanding of mathematics, from an extensive literature review, it appears that there are no studies examining learners' ability to make connections among external representations in the context of proportionality. This current state is problematic, since it is not completely clear which processes learners experience when making connections among representations in the context of proportionality. Unraveling these implicit processes may help to gain greater insights about the notions learners attend to and the links they establish among multiple representations related to proportionality.

This study extends the research on connections by exploring the nature of connections prospective teachers make among external representations involving proportionality. Exploring prospective teachers' knowledge of connections among external representations in the context of proportionality is crucial in that it provides opportunities to enhance the quality of teaching and learning in mathematics. Besides, knowing about prospective teachers' processes and difficulties in connecting among representations regarding proportionality may guide mathematics educators in making pedagogical decisions such as how to teach proportions, what type of representations to use and in which order. If prospective teachers' difficulties in proportionality are not handled appropriately during teacher education programs, they will most probably keep such difficulties and convey them to their own classrooms when they start their teaching profession.

METHODOLOGY

Research Design

A qualitative case study was conducted in order to obtain in-depth information with respect to prospective middle school mathematics teachers' knowledge of connections among external representations in the context of proportionality. Using a qualitative design helps the researcher understand the cases better (Patton 1990). Besides, it helps to "investigate a contemporary phenomenon within its real life context especially when the boundaries between phenomenon and the context are not clearly evident and rely on multiple sources of evidence, with data needing to converge in a triangulating fashion" (Yin 2003, p. 13).

Participants and the Context

Purposeful sampling strategy is superior to and more relevant than random sampling strategy in qualitative research studies (Patton, 1990). Besides, it allows for relevant and information-rich cases for carrying out in-depth studies (Patton, 1987). Thus, purposeful sampling strategy was employed in the selection of the research participants.

Three research participants were purposefully selected from senior prospective middle school mathematics teachers enrolled in a state university located in the inner region of Turkey. Pseudonyms were used in place of participants' real names to keep their identity unknown (Christensen, Johnson & Turner, 2013). Two of them were female students (i.e., Bahar and Damla) and one of them was a male student (i.e., Tolga). All participants were at the age of 22. In terms of cumulative grade point averages (CGPAs), Damla was between 2.00 and 2.50, Tolga was between 2.50 and 3.00, and Bahar was between 3.00 and 3.50. Thus, Bahar was an honor student, while Damla and Tolga were satisfactory students in terms of their academic standings. Participants were chosen by taking into account their academic standing, year level, gender and department. Thus, one female student with a high academic standing (Bahar), one male student with an average academic standing (Tolga), and one female student with a relatively low academic standing (Damla) were requested to take part in the study.

The data of the study were collected towards the end of the spring semester and just before the final exams. Thus, the participants took all courses offered by the mathematics teacher education program. Prospective teachers took three types of courses as mathematics (e.g., Geometry, Linear Algebra, and Differential Equations) mathematics education (e.g., Methods of Teaching Mathematics), and general education courses (e.g., Educational Psychology and Classroom Management). Prospective teachers mainly took mathematics courses in their first and second years, mathematics education courses in their third and four years, and general education courses in all four years.

The participating prospective teachers had some exposure to the concepts of representations and proportions in the following courses offered by the mathematics teacher education program: Methods of Mathematics Teaching-I, Methods of Mathematics Teaching-II, Teaching Practicum, and The Teaching of Fractions, Ratios, and Proportions. Methods of Mathematics Teaching courses were compulsory courses and were offered to the participants in their third year. The course syllabuses revealed that the ratios and proportions were covered comprehensively for several weeks and that specific time was allocated to the teaching of concepts related to these topics. Teaching Practice was a fourth year compulsory course which enabled prospective teachers to experience actual classroom practices with the help of a mentor teacher in public schools governed by the Turkish Ministry of National Education. By this course, the prospective teachers became aware of specific mathematics topics taught in each of the grades 5-8 and became able to design and implement activities that may promote the development of these concepts. The Teaching of Fractions, Ratios, and Proportions course was also a fourth year course. However, it was an elective course and not all participants attended it. Namely, only Damla took this course. The instructor of the course explained that he focused on the following ideas when covering his lessons: definitions of fraction, ratio, and proportion concepts, relationships among these concepts, learners' possible learning difficulties and misconceptions about these concepts, the place and importance of these concepts in Turkish school mathematics curricula, qualitative and quantitative reasoning types entailed in understanding ratios and proportions, problem types related to proportions, and solution strategies related to proportional reasoning problems.

Data Collection

Data from this qualitative study were obtained through one-on-one semi-structured interviews with three prospective middle school mathematics teachers. The interview protocol consisted of two main parts: (i) general questions regarding participants' views about connections and (ii) written mathematical tasks measuring participants' knowledge of connections among external representations in the context of proportionality. General questions determined participants' views about connections. Namely, they were asked to explain the following questions: How do you define the term "connectivity"? Explain your definition. Can you give an example? Explain your example. Next, participants were administered a set of written mathematical tasks in order to obtain written responses with respect to their knowledge of connections. In these tasks, the participants were asked to translate among tabular, symbolic, and graphical representations (see Appendix). Besides, they were asked to explain whether there exists a linearity and proportionality between the variables x and y in the given representations. Each participant was interviewed once a week and it took 45 minutes to complete each interview and data were collected in three weeks.

Prior to the main study, the semi-structured interview protocol underwent a quality review to improve its validity. The protocol was evaluated by one mathematician and one mathematics educator in a middle school mathematics teacher education program in terms of the following aspects: appropriateness of the interview questions and the written tasks in relation to the aims of the study, the use of appropriate mathematical terminology, the clarity of mathematical statements, content accuracy, and design quality. The feedback and comments provided by the expert reviewers (i.e., the mathematician and the mathematics educator) were examined by the researcher and later necessary modifications were made to enhance the quality of interview questions and written tasks. For example, the experts suggested using both linear and non-linear situations for all types of external representations. Besides, they suggested using almost or exactly the same relationships for the symbolic, tabular, and graphical representations to be able to observe the effect of different representation types on prospective teachers' understanding of proportionality. Thus, the first written task included the tabular forms of the following mathematical expressions: $y = 2x$, $y = 2x + 1$, and $y = x^2$. The second written task included the tabular forms of the following mathematical expressions: $y = -2x$, $y = 2x + 4$, and $y = 2x^2$. Finally, the third written task included the following symbolic expressions: $y = 5x$, $y = 2x + 3$, and $y = x^2$. Before the main study, pilot interviews were also conducted with another three prospective middle school mathematics teachers to increase the validity of the instruments. The pilot study participants helped the researcher improve the clarity of the statements included in the interview protocol. Besides, the researcher gained some more experience and familiarity in conducting interviews.

The interview and written task data were coded independently by the researcher and a graduate student in mathematics education. Inter-rater reliability was computed to check the rate of agreement between the two coders. There was a 95% agreement between the two coders. The conflicts between the two coders were resolved in a meeting when they discussed their individual coding and their opinions related to them.

All interview sessions were video-taped for transcription purposes. To avoid noise and interruptions, the interview was conducted in the meeting room of the faculty. Before the interview, the participants were informed about the purpose of the study and voluntary informed consent was obtained from the participants by having them sign a written statement.

During data collection and analysis process, the researcher tried to make sure that the findings and the interpretations were credible. The data of this study were obtained from several different sources to establish triangulation. Triangulation is "the process of corroborating evidence from different individuals, types of data, or methods of collection" (Creswell, 2008, p. 259). Thus, triangulation contributed to the trustworthiness of the data (Denzin, 1988; Glesne & Peshkin, 1992; Merriam, 2001). Triangulation allows the researcher to examine his or her own material critically, test it, discover its weaknesses, and decide on possible changes that would become compulsory as the study proceeds

(Fielding & Fielding, 1986). Hence, the data obtained from this study were triangulated by interviewing, videotaping, note taking, and obtaining written responses. Moreover, member checking was done to determine whether the researchers' interpretations of data were accurate or not.

Data Analysis

Data collection and analyzing are simultaneous activities in qualitative research and data analysis in a case study research aims to provide intensive and holistic description of the case (Yin, 2003). The interview data were transcribed verbatim. After the transcription process, the researcher read the whole data a few times and later the coding process began. The codes started to emerge from the data after reading and thinking about it. After generating many codes, the researcher organized them into a number of fewer categories. Finally, the researcher tried to arrive at common themes by organizing the categories. Namely, the researcher organized the categories into a smaller number of central themes. The data analysis went on until the researcher reached a logical saturation point.

RESULTS

The results of this study were reported under two main titles as participants' views about connections and participants' knowledge of connections among external representations in the context of proportionality. Participants' views about connections are presented below.

Participants' Views about Connections

The findings of this study showed that the prospective teachers viewed connections mainly as a link between different mathematical topics or concepts, as a link between mathematics and daily life, and as a tool for improving students' understanding of mathematics. Participants' views about connections are presented in more detail in the following parts.

Connections as a Link between Different Mathematical Topics

The findings of this study showed that the participants viewed connections as a tool for linking two or more different mathematical topics. For instance, Damla perceived making connections in mathematics as a transition between two different topics such as numbers and proportions. Her views about connections are exemplified as follows:

> Damla: I think connections are transitions between two different topics. For instance, when learning proportions we use integers or rational numbers. We have to use numbers when learning exponents, radicals, factorization or at least when doing four basic operations such as addition, subtraction, multiplication or division.

Tolga's ideas about connections revealed the view that students built new understandings on previous knowledge. He also reinforced that previously learnt topics were a prerequisite for learning new topics. The following excerpt illustrates this view:

> Tolga: I think connections are linking two mathematical topics to each other. Students have to know exponents in order to learn radicals. Also, in order to learn how to write fractions in their simplest forms we should know multiplication and division.

Bahar also viewed connections as linking mathematical concepts or topics. However, she stipulated that similarity of concepts is essential in order to make connections among them. Her idea is exemplified in the following vignette:

> Bahar: I think we can link two concepts to each other by means of connections. For example, in geometry classes we can teach geometrical shapes by linking them to each other. Before teaching triangles, we can talk about the properties of squares and reinforce that the diagonal divides the square into two equal triangles. Shortly, we can link everything to each other when those things have similar features.

As can be understood from the aforegiven dialogues, Damla viewed connections as a transition between two topics. Tolga emphasized that without prior knowledge, it is impossible to make connections with new knowledge. Bahar thought that similarity of concepts was mandatory for making connections. Participants' views about connections as a link between mathematics and daily life are presented below.

Connections as a Link between Mathematics and Daily Life

The findings showed that participants also viewed connections as a link between mathematics and daily life. For instance, Damla indicated that people should use and notice geometry in their daily lives. See the following example:

> Damla: We make connections in our daily lives. When we encounter a problem we think for a while and then try to correlate the problem with our past experiences. When building a house the columns take the form of a rectangle.

Bahar and Tolga expressed that connections are used very frequently in daily life. Bahar expressed that "We use connections at school, at home… Shortly, in every moment of our lives we may be making connections". Similarly, Tolga explained his idea in this way: "Whether we are aware or not, we use connections in every realm of life".

In short, Damla emphasized using geometry knowledge in finding solutions to the problems encountered in daily life, while Bahar and Tolga put more emphasis on the frequent use of connections in every aspects of life. Prospective teachers' views about connections as a tool for improving students' understanding of mathematics are presented below.

Connections as a Tool for Improving Students' Understanding of Mathematics

The findings showed that the participants also paid considerable attention to the benefits of connections on students' understanding of mathematics. For instance, Damla stressed that connections ease teaching mathematics. She expressed her view as follows:

> Damla: Connecting mathematics to daily life is important since mathematics is difficult for middle school students due to its abstract nature. By connecting mathematics to students' daily lives, mathematics will be more concrete and it will provide students long-lasting learning.

Tolga stressed the facilitator role of connections. In addition, he indicated that connections help students learn mathematics conceptually. The following excerpt illustrates this view:

> Tolga: Connections facilitate learning. Without connections, students will be forced to rote learning. After some time, they will be fed up with the way they are taught.

Bahar expressed that using connections during teaching paves the way for a robust understanding of mathematics. The following statement illustrates her idea:

> Bahar: Connections absolutely have some positive impact on learning. Current middle school mathematics curriculum is based on student-centered learning and this type of learning entails connecting mathematical ideas for deeper and more lasting understanding.

To summarize, Damla believed that connections help in concretizing abstract mathematical concepts. Tolga articulated that connections help students develop relational understanding and added that the absence of connections would force students to rote learning. Bahar argued that student centered learning approach expects students to make connections among mathematical ideas and that connections help students gain a deeper and long lasting mathematical knowledge. Thus far, findings related to prospective teachers' views about mathematical connections are presented. In the following parts, participants' knowledge of connections among external representations in the context of proportionality is presented.

Participants' Knowledge of Connections among External Representations in the Context of Proportionality

The findings related to participants' knowledge of connections among external representations in the context of proportionality are reported under three headings as connections among symbolic and tabular representations, connections among tabular and graphical representations, and connections among symbolic and graphical representations. Participants' knowledge of connections among tabular and symbolic representations is presented below.

Participants' Knowledge of Connections among Tabular and Symbolic Representations

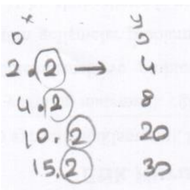
The findings showed that all participants were able to write symbolic forms of tabular representations. However, only Tolga and Bahar made necessary connections among different concepts when shifting from tabular to symbolic representations. Although Damla could write symbolic forms of tabular representations, she was doing this by rote since she could not make necessary connections related to the constant of proportion and slope. Also, she asserted that in each of the given tabular representations there was a proportionality between x and y. Therefore, this showed that she lacked some knowledge about proportions. Besides, she could not connect linearity to proportionality. For instance, the following dialogue demonstrates her responding to Task 1b.

> Interviewer: In the table above, the variables x and y have different values. By the help of this table, how can you write x and y in a symbolic form?
> Damla: In the equation y=ax+b, if x=0 and y=1 then 1=0+b, b=1. Next, we substitute y=3, x=1 and b=1 into the equation to find a. Since 3=a.1+1, then a=2. Thus, we can write the equation as y=2x+1.
> Interviewer: How did you decide that the equation is in the form of y=ax+b?
> Damla: I examined whether x and y values satisfied the equation. Since y=2x+1 is satisfied by each value of x and y, the equation is correct.
> Interviewer: By considering the values given in table can we say that y is proportional to x?
> Damla: Yes, because as x increases, y also increases. Therefore, they are proportional to each other.

Tolga could write symbolic forms of tabular representations and he was able to compute the constant of proportion in order to determine the relationship between variables. He could generally make necessary connections between tabular and symbolic representations and his knowledge of proportions was adequate. Here is a dialogue that demonstrates his responding to Task 1c.

> Interviewer: In the table above, the variables x and y have different values. By the help of this table, how did you write x and y in a symbolic form?
>
> Tolga: First, I examined whether the values of y are found by multiplying the values of x by an integer. But, I could not find such an integer. However, I realized that each value of y is the square of each value of x. Therefore, I can write the equation as $y=x^2$.
>
> Interviewer: By looking at the table can you say that x and y are proportional to each other?
>
> Tolga: No, because we do not have a constant of proportion since $\frac{y}{x}=\frac{0}{0}\neq\frac{1}{1}\neq\frac{25}{5}\neq\frac{121}{11}\neq\frac{169}{13}$. Therefore, there is not a proportionality between x and y.

Bahar conceived that equations written in the form of y=ax+b did not portray a proportional relationship since there was not a constant of proportion. Besides, she knew that a nonlinear equation did not denote a proportional relationship for x and y. Furthermore, she could distinguish the slope from the constant of proportion. The following dialogue demonstrates her responding to Task 1a.

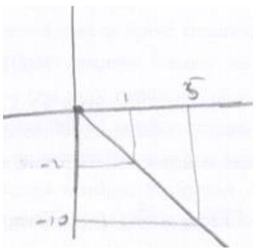
> Interviewer: In the table above, the variables x and y have different values. By the help of this table, how can you write x and y in a symbolic form?
>
> Bahar: The x values are multiplied by 2 and y values yielded. We can see this below:
>
> 
>
>
> Therefore, we can write the equation as y=2x.
>
> Interviewer: Well, what does 2 stand for?
>
> Bahar: It is the constant of proportion since we can write the equation in the form of $\frac{y}{x}=2$ and x and y are proportional to each other.
>
> Interviewer: Good, what does 2 stand for when we write the equation in the form of y=ax+b?
>
> Bahar: Aha! It is the slope.
>
> Interviewer: Is there a relationship between the constant of proportion and the slope?
>
> Bahar: Yes, they have the same values for this equation.

In brief, the prospective teachers had some difficulties when making connections among tabular and symbolic representations. For instance, Damla misconceived that all types of monotonically increasing graphs depicted a proportional relationship between the variables x and y. Tolga could determine the existence or non-existence of a direct proportion between x and y in quadratic relationships, only after by making some calculations and examining whether a constant of proportion exists. Bahar appeared to have some difficulty in connecting the constant of proportion with the slope. Namely, she seemed to have limited knowledge about the idea that the constant of proportion and the slope refer to the same notions in the relationships in the form of $y = ax$. Participants' knowledge of connections among tabular and graphical representations are presented below.

Participants' Knowledge of Connections among Tabular and Graphical Representations

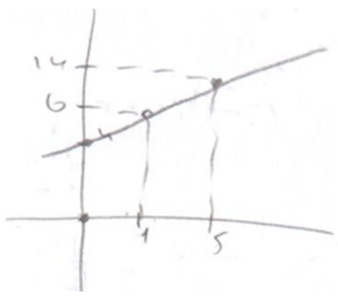
The findings showed that all participants could correctly draw the graph of the relationship between x and y by using tabular representations. While drawing the graph, Damla first shifted to the symbolic form of the tabular representation and identified the constant of proportion. She used the constant of proportion as the slope of the graph. The following dialogue demonstrates her responding to Task 2a.

> Interviewer: By using this table, how do you draw the graph denoted by x and y?
>
> Damla: I use several values of x and y and draw the graph. First, I need to discover the relationship between x and y and then write this relationship symbolically. When we examine the table, we can see that the y values can be obtained by multiplying the x values by -2. Then, $y = -2x$ depicts the relationship between x and y.
>
> Interviewer: Okay. How do you draw the graph?
>
> Damla: Here, -2 is the constant of proportion and it is the slope of the line as well. The curve of the graph is linear. Besides, the graph shows that there is a direct proportion between x and y.



Tolga could directly draw the graph without shifting to the symbolic form. He first plotted the given values of x and y on the coordinate plane. When deciding whether a curve or a straight line connects these points to each other, he used the constant of proportion and the slope. The following dialogue demonstrates his responding to Task 2b.

> Interviewer: By using this table, how do you draw the graph denoted by x and y?
>
> Tolga: Initially, I specify the location of x and y values on a coordinate plane. Later, I examine the constant of proportion or the slope of the relationship depicted by x and y values. Here, there is not a constant of proportion. However, $\frac{6-4}{1-0} = \frac{14-6}{5-1} = 2$. Thus, I obtain a graph whose slope is equal to 2 and I draw this graph in the following way:



Despite not explicitly writing the symbolic form, Bahar verbally explained the relationship between x and y and used this explanation when shifting from tabular to the graphical representation. Besides, she was able to notice that the graph was a curve in that there was a quadratic relationship. See the following dialogue for her responding to Task 2c.

Interviewer: By using this table, how do you draw the graph denoted by x and y?

Bahar: Here, the y values were obtained by first squaring x values and then multiplying them by 2. I can explain this to you in this way:

$1^2 . 2 = 2$

$5^2 . 2 = 50$

$(-5)^2 . 2 = 50$

$(-10)^2 . 2 = 200$

Interviewer: Well, what does 2 stand for in your explanation?

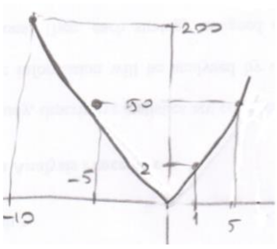
Bahar: Nothing

Interviewer: May it be a constant of proportion?

Bahar: No, it is not.

Interviewer: Why do you think so?

Bahar: Because, $\frac{y}{x} = \frac{2}{1} \neq \frac{50}{-5} \neq \frac{200}{-10}$. Besides, there is a quadratic relationship here. Thus, we can draw the following graph:



As the aforementioned dialogues show, Damla first shifted to the symbolic form of the given tabular representation in order to draw its graph and to interpret whether there exists a proportionality between x and y, while Bahar did not explicitly shift to the symbolic form. However, from her explanations it appeared that she first shifted to the symbolic form mentally. Unlike Damla and Bahar, Tolga directly plotted the points on the graph by using the tabular representation. When determining whether the curve passed through the origin and whether it denoted a proportional relationship between x and y, Damla and Tolga attempted to determine the slope and the constant of the proportion by making some calculations. Bahar indicated that there was a quadratic relationship between x and y and she roughly drew a curve when drawing the graph. However, she did not make any explanation that showed that she knew conceptually why quadratic relationships depicted non-linear situations. Besides, she did not attempt to determine either the slope or the constant of proportion. Participants' knowledge of connections among symbolic and graphical representations are presented below.

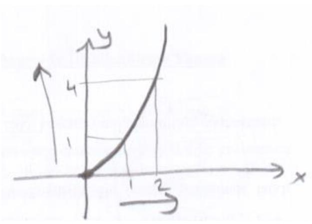
Participants' Knowledge of Connections among Symbolic and Graphical Representations

The findings showed that the participants could shift from symbolic to graphical representations correctly. However, only Tolga and Bahar could correctly interpret that lines that do not pass through the origin do not portray proportional relationships. Although Damla could easily make transitions between symbolic and graphical representations, she could not connect linearity to proportionality. She could distinguish linear relationships from nonlinear ones. However, she had some misinterpretations when deciding whether a graph portrayed a proportional relationship or not. The following dialogue demonstrates her responding to Task 3c.

Interviewer: How can you draw a graph for the equation $y=x^2$?
Damla: I give numerical values for x and find the values of y. Next, I draw a table in the following way:

x	0	1	2
y	0	1	4

Here, for x=0, y=0; x=1, y=1 and x=2, y=4. We can draw a graph for this equation in the following way:



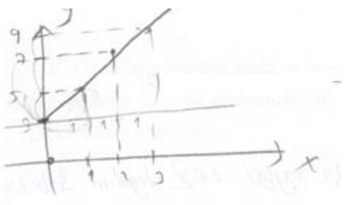
Interviewer: Is the curve passing through the origin?
Damla: Yes. But it is a parabola and there is a nonlinear relationship between x and y.
Interviewer: Do you think that x and y are proportional to each other in $y=x^2$?
Damla: Yes, because as x increases, y also increases.

Tolga could translate among symbolic and graphical representations appropriately and he was aware that a quadratic equation portrayed neither a linear relationship nor a proportional relationship. Initially, he assumed that a graph would portray proportionality although the line did not pass through the origin. However, as he contemplated more on the given task, he realized that the line had to pass through the origin in order for a graph to portray a proportional relationship. The following dialogue demonstrates his responding to Task 3b.

Interviewer: How can you draw a graph for the equation $y=2x+3$?
Tolga: I give numerical values for x and find the values of y. A table can also be drawn in the following way:

x	y
0	3
1	5
2	7
3	9

Next, I draw the graph of y=2x+3 by using the table. In that case, the graph can be drawn in the following way:



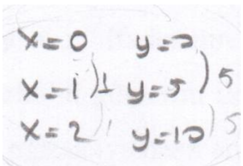
Interviewer: Do you think that x and y are proportional to each other in y=2x+3?

Tolga: Yes… But, just a moment, let me think for a while… No! I cannot rewrite the equation as $\frac{y}{x}=k$. Therefore, there is not a constant of proportion. However, the ratio of change in y to change in x is constant since $\frac{5-3}{1-0}=\frac{7-5}{2-1}=\frac{9-7}{3-2}=2$. This constant gives us the value of the slope of the line.

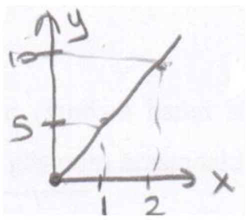
Bahar did not have any difficulty in drawing graphs of symbolic expressions. Initially, she was not certain about proportionality in a graph when the line did not pass through the origin. However, as she contemplated more on the tasks, her explanation about the linearity and the proportionality of graphs became clearer. The following dialogue shows her responding to the Task 3a.

Interviewer: How can you draw a graph for the equation y=5x?

Bahar: I give numerical values to x and find y. I do this in this way:



Bahar: Here, 5 times x is equal to y. Thus, 5 is our constant of proportion. I draw the graph in this way:



Interviewer: Well, how do you understand whether x and y are proportional to each other in y = 5x?

Bahar: As x increases, y also increases. However, the increase in y is always 5 times of x.

Interviewer: Is there always a proportional relationship between the variables if y increases as x increases?

Bahar: No, it is insufficient to explain in that way. There must be a multiplicative relationship between the increase in y and the increase in x.

To summarize, all prospective teachers were able to translate among symbolic and graphical representations. Besides, all of them shifted to the tabular form of the symbolic expressions before drawing their graphs. Although Damla was aware that there was a nonlinear relationship between the given x and y values, she considered the relationship between x and y as proportional. It seemed that "the simultaneous increase or decrease in x and y" was a guiding criterion for Damla in deciding whether the given relationships are proportional or not. Tolga also had difficulty interpreting proportionality through inspecting graphical representations. More precisely, he was indecisive in linear graphs in which the straight line did not pass through the origin. It seemed for him that linearity between two quantities was the only criterion for the proportionality of these quantities. Bahar was aware of the fact that "the simultaneous increase or decrease in x and y" was not the only requirement for proportionality. Namely, she articulated that the increase y should be a multiple of the increase in x.

DISCUSSION AND IMPLICATIONS

This study examined prospective middle school mathematics teachers' views about mathematical connections and their knowledge of connections among symbolic, tabular, and graphical representations in the context of proportionality. The findings of the study suggested that prospective teachers viewed connections mainly as a link between different mathematical topics or concepts. Participating prospective teachers' awareness of the unified nature of mathematics is important. Making connections among mathematical concepts requires conceptual understanding (Eli, Mohr-Schroeder & Lee, 2011) in that conceptual understanding requires building a network of relationships among various concepts (Hiebert & Lefevre, 1986; Johnson, Seigler & Alibali, 2001). Without noticing the role of connections in mathematics, it would be difficult for prospective teachers to understand that mathematics is not "a set of isolated facts and procedures" (NCTM, 2009, p. 3) and that understanding mathematical concepts, facts, or procedures require integrating or connecting prior knowledge with new knowledge (Eli et al., 2011).

The findings of the study suggested that prospective teachers viewed connections as a link between mathematics and daily life. This finding is in agreement with the previous studies (Gainsburg, 2008; Lee, 2012). For instance, Lee (2012) found out that prospective teachers were aware of the importance of using real-world connections and they strongly believed that real world problems should be used in teaching of mathematics. Similarly, in-service secondary school mathematics teachers participated in Gainsburg's (2008) study indicated the importance of real life connections in teaching of mathematics. They explained that they used connections when presenting word problems, planned examples or when students were working on a project. However, these in-service teachers also explained that they had difficulty in making connections due to the following constraints: connections are time consuming, they require some training to achieve effectiveness, and they are not emphasized in the curriculum. Daley and Valdes (2006) found out that most of the teachers made little or no effort to connect mathematics to learners' daily lives. Thus, the participating prospective teachers' noticing of the link between mathematics and daily life was considered promising. Real world connections have many advantages for students such as improving comprehension of mathematical concepts (De Lange, 1996), motivating learning of mathematics (National Academy of Sciences, 2003), and applying mathematics to problems confronted in real life (National Research Council, 1998).

The prospective teachers participated in this study also believed that connections serve as a tool for improving students' understanding of mathematics. The research results regarding beliefs about using connections in the teaching of mathematics appear conflicting. In some research, participants reported that mathematical connections can be used to motivate students' learning, draw their attentions to the topic, and make concepts easier for students to understand (e.g., Gainburg, 2008), while other research revealed that many teachers believed that real-world problems distracted students' attention away from the idea on which they need to concentrate (e.g., Chapman, 2006; Verschaffel, Greer & De Corte, 2000) and that prospective teachers had a tendency to abandon real life approaches and favor non-realistic ones when solving word problems (e.g., Verschaffel et al., 2000). However, mathematics education community supports the view that understanding mathematics requires making connections with real life situations (e.g., Ji, 2012; Van den Heuvel-Panhuizen & Wijers, 2005). For instance, Realistic Mathematics Education approach pays considerable attention to the use of real-life contexts in school mathematics curriculum. Thus, the participating prospective teachers' positive viewpoints towards using real life connections in the teaching of mathematics may provide them with greater advantages of connections after they start their teaching profession and consequently promote their students' higher order thinking in mathematics.

Another aim of the current study was to examine prospective teachers' knowledge of connections among symbolic, tabular, and graphical representations in the context of proportionality. The findings showed that all participants were able to translate correctly among symbolic, tabular, and graphical representations. However, not all of them performed these translations conceptually. Namely, the participating prospective teachers used their procedural knowledge when translating among representations. For instance, Bahar did not provide any evidence that shows that she knows conceptually why quadratic relationships depict a curve in a graph. All she could do was reciting that "quadratic relations do not involve straight lines in their graphical representations". A similar result was found by Eli et al. (2011). The researchers investigated the type of mathematical connections prospective middle grades teachers made while engaging in card-sorting tasks. They found out that the participants used categorical, procedural, characteristics/property, derivational, and curricular connections. However, their participants mainly used categorical (i.e., using surface features mainly to define a category) and procedural (i.e., linking mathematical ideas through procedures or algorithms). The researchers attributed prospective teachers' predominant use of categorical and procedural connections to the traditional curriculum that emphasized procedural rather than conceptual understanding of mathematics.

The participating prospective teachers of the current study might have mainly used their procedural knowledge on similar grounds. They might have developed this type of knowledge when preparing for the university entrance examination. Namely, in this examination, high school graduates complete a large number of multiple choice tasks in a limited time. In multiple choice tasks, the students are not expected to explain the relationships between mathematical concepts. Thus, the participants might have become familiar with translation tasks and they might have memorized some procedures to be able to solve such tasks. Besides, the mathematics courses in their teacher education program might have resulted in mathematical experiences that emphasized procedural fluency rather than conceptual understanding. However, conceptual understanding of the ideas that are inherent in translating among different representations is crucial for prospective teachers. Many research studies emphasize the importance of understanding multiple representations conceptually in improving mathematical understanding (Even, 1998; Lesh et al, 1987). Similarly, translating among different representations is a crucial skill for prospective teachers. Because, teaching a specific concept by using its different representations helps students build a mental network of that concept and consequently helps them develop a more profound understanding (Van de Walle, Karp & Bay-Williams, 2016). Multiple representations may be regarded as means for communicating effectively, learning meaningfully, and for having conceptual understanding (Kamii, Kirkland & Lewis, 2001). Besides, they foster students` recall and comprehension of the material (Klein, 2003).

The findings of the current study showed that not all prospective teachers could connect the given representations with the concept of proportionality. Namely, they experienced some difficulties while connecting representations to the concept of proportionality. For instance, although Damla could write symbolic forms of tabular representations correctly, she could not make necessary connections related to the constant of proportion and slope. It seemed that she was doing this transition by rote since she asserted that in each of the given tabular representation task there was a proportionality between x and y. Besides, she could not correctly interpret that lines that did not pass through the origin did not portray proportional relationships. This showed that she had some misinterpretations when deciding whether a graph portrayed a proportional relationship or not. This finding appears to be in line with the findings of Debock et al. (2017). They examined students' rate of accuracy in linking functions to their properties and attempted to uncover whether this accuracy depended on the representational mode of functions. They found out that graphical representations were less supportive for students than symbolic and graphical representations. The participating teachers' difficulty in interpreting proportionality in graphical representations might be explained by the fact that they might have been less exposed to graphical representations in mathematics classes compared to the symbolic and tabular ones. Knuth (2000) explored high school students' understanding of the connections between algebraic and graphical representations of functions. He expressed a similar sentiment that over seventy five percent of the high school students participated in his study selected an algebraic approach as their main solution method, even in cases where a graphical approach seemed to be simpler and more productive than the algebraic one. He found this result distressing since more than half of the students had taken several mathematics courses as first-year algebra, geometry, second-year algebra, pre-calculus, and advanced placement calculus.

The current study was conducted in order to contribute to the literature about prospective middle school mathematics teachers' knowledge and understanding of connections among representations in the context of proportionality. This study uncovered prospective teachers' thinking processes about the concept of proportion in greater depth through semi-structured interviews. However, the written tasks administered to the prospective teachers in this study were limited to linear proportional situations in which one quantity (i.e., the variable x) is directly proportional to another quantity (i.e., the variable y). Apart from linear proportional relationships, there are also non-linear proportional situations in which a quantity is directly proportional to the square, cube or any other power of another quantity (e.g., the relationship between y and x^2 in $y = x^2$) (Ayan & Işıksal Bostan, 2016). Students experience non-linear proportional situations frequently not only in mathematics but also in other domains such as science and engineering. For instance, the area formula of a circle is $A = \pi r^2$ and it suggests that the area of a circle is directly proportional to the square of its radius. Similarly, in science, the free fall formula (

$h = \frac{1}{2}gt^2$) suggests that the square of time elapsed is directly proportional to the displacement of the falling object. Kooij and Goddijn (2011) argues that, despite its significance, such multifaceted uses of proportionality are not adequately addressed in the teaching of algebra. Thus, future research may focus on prospective middle school mathematics teachers' knowledge of connections in the context of non-linear proportionality and may uncover their strategies and possible difficulties confronted in solving non-linear proportional problems. From this respect, the findings of the current study may give some inspiration to the researchers who are interested in exploring prospective teachers' knowledge of non-linear proportionality.

The mathematics and mathematics education courses offered to the prospective teachers through teacher education programs may adopt an approach that emphasizes the importance of mathematical connections. These courses may be reorganized in a way that focus on conceptual understanding through using connections. Besides, new elective courses may be designed in teacher education programs in order to have prospective teachers gain more experience with mathematical connections. Adu-Gyamfi et al. (2007) suggested instructors to use tasks that force reversibility, flexibility, and generalizability in their courses to cover functions more effectively and to shed light on students' misconceptions about connections among representations. They further added that standard instructional practices do not help to remedy limitations in prospective teachers' knowledge of representation system, knowledge of domain, and knowledge of domain register. Thus, mathematicians and mathematics educators may consider the aforementioned three key constructs in their courses and uncover prospective teachers' limitations in their knowledge of representations and proportionality and help future researchers understand the larger picture of representations and proportionality.

Besides, instructional materials may be designed for prospective teachers to attract their attention to the role and importance of connections in the teaching and learning of mathematics. For instance, textbooks that provide rich variety of good connections may be prepared for prospective teachers in that such textbooks may be used as a main source for ideas about connections.

Ultimately, in-service teachers may not have adequate content knowledge related to connections as well. Bearing this in mind, professional development activities can be organized to increase in-service teachers' awareness and knowledge of mathematical connections and the notion of proportionality. However, Orrill and Kittleson (2015) reported that professional development programs may fall short of developing teachers' content knowledge for teaching if they are not "designed to make knowledge resource connections apparent in ways that allow teachers to explicitly adopt or reject content and pedagogy rather than leaving their use of those resources tacit" (p. 294). Thus, specific attention should be devoted to the designing of professional development programs in order to help teachers translate connection making moves readily into their actual classroom practices.

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APPENDIX

1. The tabular representations given below depict relationships between the variables x and y. Write each relationship in a symbolic form. Explain whether each relationship is proportional or not.

a)

x	y
2	4
4	8
10	20
11	22
15	30

b)

x	y
0	1
1	3
5	11
11	23
13	27

c)

x	y
0	0
1	1
5	25
11	121
13	169

2. The tabular representations given below depict relationships between the variables x and y. Draw the graph of each relationship. Explain whether each relationship is proportional or not.

a)

x	y
1	-2
5	-10
-5	10
-10	20

b)

x	y
0	4
1	6
5	14
-5	-6
-10	-16

c)

x	y
0	0
1	2
5	50
-5	50
-10	200

3. The symbolic representations given below depict relationships between the variables x and y. Draw the graph of each relationship. Explain whether each relationship is proportional or not.

a) $y = 5x$

b) $y = 2x + 3$

c) $y = x^2$